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NUMERICAL PROCEDURES FOR THE CALCULATION OF THE STRESSES IN MONOCOQUES

I - DIFFUSION OF TENSILE STRINGER LOADS IN REINFORCED PANELS

By N. J. Hoff, Robert S. Levy, and Joseph Kempner
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SUMMARY

Experiments were carried out at the Polytechnic Institute of Brooklyn with both curved and flat reinforced sheet models the longitudinals of which were loaded axially. The stress distribution in longitudinals and sheet was measured with electric strain gages. The stresses then were calculated with the aid of a procedure of successive approximations based upon simplifying assumptions concerning the state of stress in a simple reinforced panel. The agreement between calculations and experiment was found to be reasonably good.

INTRODUCTION

The methods of and the formulas used in the analysis of monocoque aircraft structures have been developed almost invariably for cylinders of circular, or possibly elliptic, cross section and of uniform mechanical properties. Yet in actual aircraft such structural elements are seldom if ever found. Unfortunately, the direct methods of analysis are little suited to cope with the problems involving complex cross-sectional shapes, irregular distribution of reinforcing elements, concentrated loads, and cut-outs. It is believed that the indirect methods recently advanced by Hardy Cross (reference 1), and particularly by R. V. Southwell (reference 2), promise a solution of such problems.

In this indirect approach the stress distribution in a structure under specified loads is determined through step-by-step approximations. In each step the state of distortion of the structure is arbitrarily modified and the stresses corresponding to the distortion are calculated. The procedure must be continued until the stresses and the external loads over the entire structure are in equilibrium. When the steps are undertaken at random, the procedure is likely to lead to a solution only, if ever, after a very great number of steps. If the calculations are to be well convergent - that is, if a reasonably rapid approach to the final state of distortion is to be attained - the steps must be undertaken according to suitable predetermined patterns. This is the reason Southwell called the procedure "Method of Systematic Relaxations."

It is the object of the present investigations to develop patterns which make a solution possible, with engineering accuracy, through a limited number of steps. This end is approached by means of theoretical considerations, strain measurements, and comparative calculations. The immediate goal is to work out a procedure which permits the solution of the complex problems previously mentioned even though approximate results are all that may be attained for the time being.

The procedure can be refined so that it will give more accurate results. It is planned to carry out this development after the more immediate problems are solved.

In this first report experiments are described which were performed in the Aircraft Structures Laboratory of the Polytechnic Institute of Brooklyn with both curved and flat sheet-stringer combinations. For his contribution to the development of the apparatus and the testing technique, credit is due to Albert J. Cullen. The stress distribution under concentrated loads was investigated with the aid of Baldwin-Southwark Metalectric strain gages. Displacement patterns were developed for the step-by-step procedure the use of which permits a rapid convergence of the computations. The results of the calculations were in reasonably good agreement with the tests.

The report is presented so that it can be understood without a previous knowledge of the Southwell or the Hardy Cross method.

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SYMBOLS

b	distance between adjacent longitudinals
h	distance between adjacent transverse reinforcements
t	thickness of sheet
v	vertical displacement
v_{block}	block displacement
v_N	vertical displacement of point N
$v_{n\ tot}$	total vertical displacement of point N
x, y	coordinates
$\widehat{yy}_{MN}$	influence coefficient
A_{tot}	total effective cross-sectional area of a stringer
$A_{tot\ cent}$	total effective area of a central stringer
$A_{tot\ edge}$	total effective area of an edge stringer
A-K	symbols used to designate horizontal sections through curved specimen
A-Z	symbols used to designate points of intersection of longitudinal and transverse reinforcements
B, M, T	symbols used to designate bottom, middle, and top horizontal sections, respectively, through flat specimen
C'	location of point C after displacement
E	modulus of elasticity

F	tensile force in bar
F	designation of a load condition of the curved specimen
G	modulus of elasticity in shear
H, J	designations of load conditions of the curved specimen
L	length
L_{mn}	distance between points M and N
V	shear force in panel
X_n	horizontal force at point N
Y_n	vertical force at point N
γ	unit shear strain
γ_{av}	average unit shear strain in panel
σ	direct stress in stringer
σ_{av}	average direct stress in a horizontal section of the sheet
τ	shear stress
1-49	symbols used to designate strain gages
I-V	symbols used to designate stringers

EXPERIMENTAL INVESTIGATIONS

Experiments with a Reinforced Curved Sheet

One of the two identical semimonocoque models is shown in figure 1. It consists of a semicircular cylinder of galvanized steel sheet reinforced both in the longitudinal and circumferential directions with hot rolled steel strips. To retain the original shape of the models under load three heavy channel section "supporting rings," one each at top, center, and bottom, were fastened to each model. These rings, shown in the photographs of the test setup (figs. 2 and 3), were attached only at the central stringer and were covered with grease so that they were capable of carrying loads only perpendicular to the surface of the model.

The load was applied by an ordinary automobile jack through a system of frames and levers shown in the photograph of figure 4. This system transmitted equal loads to the bottom extensions of the central stringers of the two models. The lever system at the top was so designed as to divide the reactionary forces approximately equally among the upper extensions of the 10 stringers contained in the two models. At the same time the upper ends of the stringers were not restrained from relative vertical displacements. All movable joints were lubricated.

Since the loads applied at the top of the models were not collinear with those at the bottom, a bending moment was exerted upon each of the models. Because of the symmetrical arrangement of the two models, these moments were equal and opposite. It was consequently possible to make them balance each other through suitable connecting elements. The balancing forces were transmitted through the supporting rings. Corresponding supporting rings were connected by thin cables and turn buckles at the bottom, piano wire at the center, and a double knife edge between bearing pads at the top.

To check the load distribution and to obtain several independent indications of the load, 14 calibrated load links were used. The forces were measured with Baldwin-Southwark SR-4 Metalectric strain gages of type A-1 and an SR-4 control box. On the 14 load links as well as at the 35 reference points where the strain was measured on the model, the gages were arranged in pairs on opposite sides of the structural element, connected in series in order to measure the average direct stress. A dummy gage was provided close to the model to provide for temperature compensation.

The switching arrangement consisted of 49 brass blocks having 1/2 inch tapered holes and a brass plug. Belden No. 18 solid waxed cotton insulated push back wire was used for all wiring.

After a number of preliminary tests, the final test runs F, H, and J were carried out corresponding to total loads of approximately 1500, 4500, and 3000 pounds, respectively, equally divided between the two models. The data presented are averages of readings made for four to six load increments. The individual values differed only slightly.

Experiments with the Reinforced Flat Sheet

The test model shown in figure 5 consisted of a flat sheet of 24S-T aluminum alloy reinforced with longitudinally and transversely arranged hot rolled steel strips. The test setup is shown in the photograph of figure 6. The load was applied by dead weights through a lever system which transmitted equal forces to the bottom extensions of the four stringers. The top end of the model was attached at the extensions of the two edge stringers to an equal arm lever. To preclude the buckling of the upper edge of the specimen, the distance between the stringer extensions was maintained by two steel spreading bars. Two lugs extending from these bars provided a lubricated sliding support for the center of the upper edge of the model.

Loads and strains were again measured with Baldwin-Southwark Metalectric strain gages, the loads through four load links and two pairs of gages at the upper two stringer extensions, the strains through 30 pairs of gages attached to the model. The dummy used consisted of a square of aluminum and steel similar to a section of the model. Gages were mounted in pairs on both the sheet and the stringers. All wiring was done with No. 20 Roeplastic insulated solid copper wire.

After several preliminary test runs, the final tests were made at load increments of 240 and 480 pounds, respectively, starting from a tare load of 240 pounds. The data presented are averages of six and five test runs, respectively. The individual values differed only slightly.

ANALYSIS OF TEST RESULTS

Curved Model

Values of the loads and stresses are presented in figures 7, 8, and 9 for the three final load conditions. These figures also contain a schematic sketch of the lever system. Values of the loads were obtained through the use of the experimentally determined calibration constants, those of the stresses through the use of the bridge and gage constants furnished by the gage manufacturer. When strain was converted to stress, the modulus of elasticity was assumed to be 30×10^6 pounds per square inch, and the state of stress to be uniaxial. Comparison of the individual load link readings permitted a check of the accuracy of the load measurement. The maximum deviation was 4.57 percent.

The variation of the tensile stress in the stringers is shown in figure 10 for the 3000-pound load condition. The distributions for the other two conditions were similar and are omitted here. The variation of the direct stress in transverse sections of the model is given in figures 11 and 12 corresponding to the 3000 and 4500 pound load conditions, respectively. It may be seen that although the curves are rather jagged they are consistent for the two cases shown. The curves for the third condition are quite similar and for this reason are not presented.

The shape of the curves justifies the use of the conception of the effective width since the values of the stress in the centers of the panels are materially lower than those close to the stringers. The magnitude of the effective width of sheet was determined by multiplying the total width of the sheet by the ratio of the average sheet stress to the weighted average stringer stress. In weighting the stringer stress the central stringers were counted twice, the edge stringers once, corresponding to the number of adjacent effective strips of sheet. The results of these calculations are as follows:

	Effective width at:	
Run	Section K (in.)	Section G (in.)
F (1500 lb)	9.72	16.50
J (3000 lb)	8.11	15.30
H (4500 lb)	9.33	17.60

In the calculations by successive approximations, which were carried out for the 3000 pound load condition, average values were used for the effective width. From the measured values the total effective area of a central stringer was found to be 0.155 square inch, that of an edge stringer 0.140 square inch. Moreover, a check on the accuracy of the stress measurements was possible, since the total load carried by any horizontal section across the model must equal the applied load. This check gave a maximum error of 8.27 percent. It was also found that the load carried by the sheet averaged 12.8 and 22.5 percent of the total load in sections K and G, respectively.

The distribution of the shearing stress in the sheet was also calculated from the test data. In this calculation it was assumed that the shear strain was constant across each panel and equal to the relative displacement of the central points of adjacent stringer segments. The angle of shear was first determined from the displacements of these points on the stringers considering the upper end point of each stringer fixed in its original position before loading. The vertical displacement of any point of a stringer could be calculated with the aid of a graphical integration of the stringer stress curve. Any relative rigid body displacement of two adjacent stringers gives rise to uniform shear strain and shear stress all along a vertical section through the model. The actual relative displacement of two adjacent stringers could be determined therefore from the condition of equilibrium of the vertical forces. The results of these calculations are presented for the 3000-pound load conditions only, since there is practically no difference between the diagrams corresponding to the different load conditions except for the scale. Figure 13 shows the shear stress along the stringers, figure 14 the deflected shape of the model.

Flat Model

The data for the flat model were analyzed in the same way as those for the curved model to obtain the loads and stresses shown on the schematic drawing of the model (fig. 15 and 16), the curves of direct stress in stringers (fig. 17) and the curves of direct stress in sheet (fig. 18). Only the curves corresponding to the 240-pound load increment case are presented here. Those corresponding to the 480-pound load increment were omitted since they are practically identical with the former ones if drawn to half the scale.

The curves of stress distribution in the sheet present a more regular appearance than do those previously shown for the curved model. This might be due to the fact that the gages on the flat model were located farther from the edges than those on the curved model.

In the evaluation of the total load carried by the sheet the effect of the overlapped portions of the sheet had to be considered. The sheet of the flat model was composed of three sections joined at the central stringers with an overlap of 1/4 inch on each side of the center lines. It was assumed that the overlapped portions were subjected to the same stress as the stringer. The effective width of sheet was calculated in the manner discussed for the curved model. Its value was found to be 7.37, 7.75, and 5.03 inches in the top, middle, and bottom sections, respectively. The average total effective area of an edge stringer was found to be 0.1301 square inch, that of a central stringer 0.1418 square inch. The latter includes the overlap. It should be noted that both for the central and the edge stringer the areas of effective width of the aluminum sheet were converted into equivalent areas of steel.

The comparison of the total load carried in a horizontal section across the model with the applied load was again made. The maximum error was 5.5 percent. The load carried by the sheet was 18.8, 21.9, and 17.8 percent of the total load measured in sections T, M, and B, respectively.

CALCULATION OF THE STRESSES BY SUCCESSIVE APPROXIMATIONS

General Features of the Procedure

In the procedure of successive approximations as developed by R. V. Southwell the stresses in an elastic structure are determined indirectly through the calculation of the elastic displacements. At the outset it is assumed that a number of points of the elastic structure are rigidly attached to an imaginary rigid body. Step by step one point after another is freed from its imaginary connections — in the language of the procedure "released" — and moved in a direction which presumably brings it closer to its final position in the loaded elastic structure. After each step the point that was moved is connected again to the rigid body, but in its new position. The forces caused in the elastic body by the displacements are calculated in each step. Through a sufficient number of steps these internal forces can be brought into equilibrium with one another and with the given external loads (accurately enough for practical purposes) without resort to imaginary forces originating from the imaginary rigid body. When this is the case, in the parlance of the procedure the elastic body is "relaxed." The displacements in this state are the actual displacements of the points of the elastic structure under the specified loads, and the corresponding internal forces are the actual internal forces caused by the specified loading, in accordance with Kirchhoff's theorem of the uniqueness of the solution of problems of elasticity.

An elastic structure can be assumed to contain an infinite number of mass points. It is obviously impossible to consider each one of them in the manner just discussed when the successive approximation procedure is applied to the structure. The procedure can be carried out, however, if the structure is imagined to be decomposed into finite "units" which, through suitable assumptions concerning their elastic properties, are considered capable of only a limited number of elastic distortions. The choice of the unit, the assumptions concerning its elastic properties, and the calculation of the forces arising from distortions of the unit constitute the "unit problem."

The Unit Problem

The unit of the elastic structure considered in this paper consists of a panel of sheet metal and the four segments of bars attached to its edges (fig. 19). The sheet is plane in one of the test specimens, and in the other circular-cylindrical with straight generatrices running parallel to line AD. It is assumed that the bars are attached to one another by ideal pins, and that they have infinite rigidity in bending. The most general distortion of the unit consists then of arbitrary displacements of the four corner points A, B, C, and D on the (plane and/or cylindrical) surface of the sheet.

The unit problem reduces, therefore, to the calculation of the forces caused by a displacement of point C in figure 19a to the position C' in figure 19b. This displacement entails the stretching of bar BC to the length $h + v$. At the same time the fibers of the sheet are also stretched. Instead of actually calculating the force required to stretch the sheet, it is preferable to take into account the resistance of the sheet to stretching through the addition of a suitably chosen effective area of sheet to the cross-sectional area of the longitudinal BC. The tensile force F required at points B and C for this deformation is then

$E = \frac{F}{A_{tot}} \frac{h}{u}$

$$F = EA_{tot}(v/h) \tag{1}$$

where E is Young's modulus of the material, and A_{tot} the cross-sectional area of the longitudinal augmented by the effective area of the sheet.

In addition to the stretching of the fibers, the displacement pattern of figure 19b incorporates slidings of the fibers relative to one another. The corresponding angle of shear varies linearly from zero at A and B to its maximum value at D and C', the average value being equal to $v/2$ divided by the width b of the panel. Consequently the average shearing stress

$$\tau = \gamma_{av}G = \frac{vG}{2b} \tag{2}$$

The total force V necessary to overcome the shear resistance of the sheet

$$V = \tau th = \frac{vGth}{2b} \tag{3}$$

where t is the thickness of the sheet. Because it is imperative to reduce the number of points where the equilibrium of forces is considered, the distributed shearing stress along bar BC is replaced by two forces, each of a magnitude $V/2$, applied at points B and C, respectively. Similarly the total shear force transmitted to bar AD is assumed to be concentrated at points A and D.

By imagining now that at first points A, B, C, and D are connected by rigid pegs to a rigid body in their original positions according to figure 19a and subsequently the peg at C is removed, point C of the elastic structure displaced to position C' through the application of the required force, and then the structure secured in its new position through the insertion of a new peg at C', it is seen that the elastic structure must be in a state of stress. Because of this it exerts forces upon the pegs the magnitude of which can easily be calculated with the aid of equations (1) to (3) and the remarks made in connection with them. The vertical components of the forces exerted by the elastic structure upon the rigid body through the intermediary of the pegs are denoted by Y and a subscript which signifies the point at which the force is acting. These components are considered positive if acting downward. The displacement v of point C is also considered positive downward. With this notation the following expressions are obtained:

$$\left.\begin{aligned} Y_A &= (Gth/4b)v \\ Y_B &= [(EA_{tot}/h) - (Gth/4B)]v \\ Y_C &= -[(EA_{tot}/h) + (Gth/4b)]v \\ Y_D &= (Gth/4b)v \end{aligned}\right\} \qquad (4)$$

The algebraic sum of the four forces Y is, of course, zero for reasons of equilibrium. At the same time the horizontal shear stress in the sheet gives rise to horizontal forces X which must also be transmitted to the imaginary pegs at points A, B, C, and D. Because of the symmetry of specimen and loading, the horizontal forces are automatically balanced in the examples discussed in the present paper. Moreover, they are small. Consequently the horizontal forces are disregarded in all the calculations to follow.

It is believed that the arbitrary assumption of an effective width of sheet and of an average shear stress preserves the salient features of the much more complex actual state of stress in the unit problem. This belief is substantiated by the reasonable agreement between experiment and the stress values calculated by the successive approximation procedure based on the present solution of the unit problem. It must be admitted, however, that the use of an effective width value derived from the tests may have contributed to this agreement. It is planned to investigate the unit problem with greater rigor as soon as the more urgent problems concerning the use of the procedure are solved.

Influence Coefficients and Operations Table

The influence coefficient $\widehat{yy}_{AB}$ is defined as the vertical downward (positive y-) component of the force which acts upon the imaginary peg (the "constraint") at A when point B is moved through a unit distance vertically downward (in the positive y-direction). In the case of the unit problem of figure 19 the multipliers of v in equations (4) are the influence coefficients. The multiplier in the first of the equations is $\widehat{yy}_{AC}$, in the second $\widehat{yy}_{BC}$, in the third $\widehat{yy}_{CC}$, and in the fourth $\widehat{yy}_{DC}$.

When the elastic structure consists of several panels, the effect of each one must be considered. Thus in the example of figure 20 a displacement v of point A causes shear stresses to occur in all the four panels. At the four corner points B, D, F, and H this circumstance does not entail any changes in the expressions for the influence coefficients derived previously, but at the midpoints C, E, G, and I of the four edge-bars the effect of the shear flow in two adjacent panels is superimposed. Accordingly,

$$
\begin{aligned}
\widehat{yy}_{BA} &= \widehat{yy}_{DA} = \widehat{yy}_{FA} = \widehat{yy}_{HA} = Gth/4b \\
\widehat{yy}_{IA} &= \widehat{yy}_{EA} = Gth/2b \\
\widehat{yy}_{CA} &= \widehat{yy}_{GA} = (EA_{tot}/h) - (Gth/2b) \\
\widehat{yy}_{AA} &= -(2EA_{tot}/h) - (Gth/b)
\end{aligned}
\qquad (5)
$$

Again the sum of all the influence coefficients is zero because of the requirements of the equilibrium of forces in the vertical direction. This fact is helpful in calculating the influence coefficient of the moving point: it is equal to -1 times the sum of the influence coefficients of the fixed points. In the tables of influence coefficients given in this report only the coefficients having two different subscripts are listed.

The operations table lists the forces that act upon the imaginary constraints because of the different "operations" undertaken. Each operation consists of the displacement of one or more points of the structure while the remaining points stay fixed. In the preceding calculations the only operations considered were those in which a single point was moved. These simple operations are always listed in the first rows of the operations table. The rapidity of the convergence of the "relaxations" discussed in the following section can be materially increased if "group" and "block" displacements are also used. A group displacement is defined as any combination of elementary displacements. A block displacement is that kind of group displacement in which the distances of two or more points are preserved, that is, in which two or more points are displaced simultaneously as a rigid block relative to the rest of the points of the structure.

In the present investigation it was found advantageous to make use of block displacements in which an entire longitudinal was moved. The forces introduced by such a block displacement can be found by adding up the forces caused by the individual displacements of each point involved. In some cases, however, it is simpler to calculate the forces directly in the same way the unit problem was solved.

As an example of a block displacement let longitudinal EFGH in figure 21 be moved downward through a unit distance $v = 1$. Since no portion of the stringers is elongated, and consequently no direct stress set up, the forces induced are:

$$\left.\begin{aligned} Y_A = Y_D = Y_I = Y_L &= Gth/4b \\ Y_B = Y_C = Y_J = Y_K &= Gth/2b \\ Y_E = Y_H &= -Gth/2b \\ Y_F = Y_G &= -Gth/b \end{aligned}\right\} \text{multiply by 2 } (\epsilon)$$

Relaxation Table and the Calculation of the Stresses

The operations listed in the operations table, multiplied by suitable constants, are entered in the relaxation table in such a way as to bring about most rapidly an approach to complete equilibrium in the procedure of successive approximations. In the first row of the table the applied loads are given. Since the loads and their reactions do not act at the same points, and since before the structure is elastically distorted in the step-by-step procedure no internal forces are assumed to act in the structure, the loads and reactions must be considered as being transmitted through the "pegs" to the imaginary rigid body. Without the imaginary restraints, therefore, no equilibrium is possible.

In the first step of the relaxations it appears advantageous to displace the point at which the greatest (unbalanced) external force is acting. For this purpose the operation should be chosen from the operations table that, while balancing the force in question, introduces the smallest possible forces at the neighboring points. When the operation is performed, the point which was displaced is in equilibrium, but a number of other points are unbalanced (if the forces transmitted by the imaginary pegs are disregarded). It seems reasonable to proceed then to the balancing of the greatest remaining unbalanced force with the aid of the most suitable operation and to continue this procedure until, after a sufficient number of steps, all the unbalanced forces are reduced to values small enough to be considered negligible for practical purposes.

The procedure just described works well when applied to simple structures in which the balancing of one point does not throw large unbalanced forces to a great number of other points. In the present problem, however, the convergence of such a procedure is very slow. The rapidity of the convergence can be increased if the operations involving simple displacements are supplemented by operations involving group displacements developed from a consideration of the most likely displacement patterns of the elastic structure.

The boundary conditions of the problems investigated in the present paper consist of given values of the forces at the ends of the longitudinals. The displacements of the end points are not restricted. Obviously the smaller one of the end loads on any single longitudinal is transmitted through the longitudinal to balance part of the larger end load, while the difference of the two end loads must be transmitted through the sheet to the neighboring longitudinals. The smaller end load, therefore, causes a uniform elongation of the stringer, while the difference of the two end loads gives rise to varying elongations of the stringer and to shearing strain in the panels of sheet. Moreover, since the force required for an elongation of the stringer is much greater than that required for a comparable displacement due to shear, the succession of steps listed below was found advantageous and followed in the balancing procedure:

(1) Displace individual points of one stringer only until the unbalanced forces along the stringer attain magnitudes approximately proportional to those given for the block displacement discussed at the end of the preceding section.

(2) Displace the stringer as a block by an amount sufficient to balance as much of these forces as possible.

(3) Repeat the steps described under (1) and (2) with the same longitudinal and the others contained in the structure until the unbalanced forces attain values which can be considered negligibly small.

The success of the procedure described here is due to the fact that steps listed under (1) cause little change in the adjacent longitudinals because of the small shear rigidity of the sheet.

When the relaxation is completed, the displacement of each point must be computed by adding up the displacements it underwent in each operation. It is advisable to list these values in a check table and to calculate from them the forces at each point with the aid of the operations table. The forces should be entered in the check table and added up. The sums of the forces are then listed in the last row of the check table. These sums may differ from those given in the last row of the relaxation table because of cumulative arithmetic inaccuracies, and possible mistakes made during the relaxations. One of the great advantages of the present procedure is that these mistakes need not be traced back and corrected in the relaxation table even if they cause sizeable unbalanced forces to appear in the last row of the check table. Instead, the unbalanced forces can be assumed as a new loading for the structure, and the relaxation can be continued until they are reduced to negligibly small quantities.

When the relaxation is completed, the stresses in the elastic structure may be computed. The direct stress in a segment of a vertical bar between points M and N is

$$\sigma = (v_N - v_M)E/L_{MN} \tag{7}$$

where L_{MN} is the length of the segment. The shear stress in the sheet between points P and Q on adjacent verticals is

$$\tau = (v_Q - v_P)G/b \tag{8}$$

where b is the distance between the verticals.

Numerical Example - The Flat Sheet Tested

The manner in which the method of successive approximations can be applied to practical problems is shown in the following example of the flat sheet described in the section on Experimental Investigations. With the aid of the effective areas of edge and center stringer calculated in the section on Analysis of Test Results and the equations previously given in the present section, the unit problem may be solved and the operations table set up. Since the model and the loading are symmetrical, shear is not transmitted by the central panel. Consequently all calculations may be based on one-half the model (fig. 22).

By using the following numerical values

$$A_{tot\ edge}E/h = (0.1301 \times 30 \times 10^6)/8 = 48.8 \times 10^4$$

$$A_{tot\ cent}E/h = (0.1418 \times 30 \times 10^6)/8 = 53.2 \times 10^4$$

$$Gth/4b = (3.8 \times 10^6 \times 0.021 \times 8)/4 \times 8 = 2.00 \times 10^4$$

the influence coefficients can be readily determined. They are tabulated as follows:

Influence Coefficients

(lb/in. × 10⁻⁴)

nm	AB	AE	AF	BE	BF	EF	EJ	EK
$\widehat{yy}_{nm}$	2.00	46.8	2.00	2.00	51.2	4.00	46.8	2.00

nm	FJ	FK	JK	JN	JO	KN	KO	NO
$\widehat{yy}_{nm}$	2.00	51.2	4.00	46.8	2.00	2.00	51.2	2.00

The operations table is obtained from the tabulated influence coefficients.

Operations Table

[Forces in lb, displacements in in. × 10⁴]

Displ.	Y_A	Y_B	Y_E	Y_F	Y_J	Y_K	Y_N	Y_O
$v_A = 1$	-50.8	2.00	46.8	2.00				
$v_B = 1$	2.00	-55.2	2.00	51.2				
$v_E = 1$	46.8	2.00	-101.6	4.00	46.8	2.00		
$v_F = 1$	2.00	51.2	4.00	-110.4	2.00	51.2		
$v_J = 1$			46.8	2.00	-101.6	4.00	46.8	2.00
$v_K = 1$			2.00	51.2	4.00	-110.4	2.00	51.2
$v_N = 1$					46.8	2.0	-50.8	2.00
$v_O = 1$					2.00	51.2	2.00	-55.2
$v_{block} = 1$	-4.00	4.00	-8.00	8.00	-8.00	8.00	-4.00	4.00

<u>Note</u>: v_{block} corresponds to

$$v_A = v_E = v_J = v_N = 1 \text{ simultaneously}$$

From the equilibrium of the model shown in figure 22 it can be seen that the 60-pound force at N is transmitted to A by direct stress in the stringer; while the 60-pound force at O is transmitted to A by shear in the sheet. If the unbalanced forces are distributed in such manner that

$$Y_A = -Y_B = Y_N = -Y_O = -10$$

and

$$Y_E = -Y_F = Y_J = -Y_K = -20$$

and if a block displacement of proper magnitude is taken, equilibrium will be attained. This procedure is followed in the relaxation table as closely as the operations table permits.

Relaxation Table

	Y_A	Y_B	Y_E	Y_F	Y_J	Y_K	Y_N	Y_O
External Loads $v_A = -2.16$	-120 110	 -4	 -101	 -4			60	60
$v_J = 1.73$	-10	-4	-101 81	-4 3	 -176	 7	60 81	60 3
$v_N = 3.33$	-10	-4	-20	-1	-176 156	7 7	141 -169	63 7
$v_B = -.25$	-10 -1	-4 14	-20 -1	-1 -13	-20	14	-28	70
$v_K = .66$	-11	10	-21 1	-14 34	-20 3	14 -73	-28 1	70 34
$v_O = 1.54$	-11	10	-20	20	-17 3	-59 79	-27 3	104 -85
$v_{block} = -2.5$	-11 10	10 -10	-20 20	20 -20	-14 20	20 -20	-24 10	19 -10
$v_J = -.04$	-1	0	0 -2	0	6 4	0	-14 -2	9
$v_N = -.25$	-1	0	-2	0	10 -12	0 -1	-16 13	9 -1
$v_B = -.02$	-1	0 1	-2	0 -1	-2	-1	-3	8
$v_K = .06$	-1	1	-2	-1 3	-2	-1 -7	-3	8 3
$v_O = .20$	-1	1	-2	2	-2	-8 10	-3	11 -11
$v_{block} = -.25$	-1 1	1 -1	-2 2	2 -2	-2 2	2 -2	-3 1	0 -1
	0	0	0	0	0	0	-2	-1

The first row of figures in the relaxation table shows the external forces with their proper signs. In order to get the desired unbalanced force at A, 110 pounds must be applied at that point. From the operations table it can be seen that this force may best be obtained by displacing point A. The magnitude of the displacement must be

$$[110/(-50.8)] \times 1 = -2.16 \text{ units}$$

The other forces caused by a unit displacement of A are multiplied by -2.16, and these values are used to fill in the second row of the relaxation table. The "residual forces" - that is, the forces remaining after a relaxation has been applied - are obtained by adding rows 1 and 2. Throughout the table, the values below the solid lines are the residual forces. In order to get the desired force at E without introducing new forces at A, point J is displaced and the calculations are carried out in a manner similar to that described in connection with the displacement of point A. This procedure is continued, and also applied to the other stringer until a block displacement appears to be advantageous. It can be seen from the relaxation table that the residual forces are close to zero after the block displacement has been made. In order to obtain more accurate values, the remaining forces are again relaxed until a new block displacement may be taken. All the residual forces are now small enough to be neglected. However, further relaxations could be made if greater accuracy were necessary.

After completion of the relaxation table the check table is set up. The sums of the forces, given in the last row of the check table, differ slightly from corresponding values in the relaxation table. This is due to the fact that fractions were neglected. However, the residual forces are small enough to be disregarded.

Check Table

v_{tot}	Y_A	Y_B	Y_E	Y_F	Y_J	Y_K	Y_N	Y_O
External loads	−120						60	60
$v_A = -4.91$	250	−10	−230	−10				
$v_B = -.27$	−1	15	−1	−14				
$v_E = -2.75$	−129	−6	280	−11	−129	−6		
$v_F = 0$								
$v_J = -1.06$			−50	−2	108	−4	−50	−2
$v_K = .72$			1	37	3	−79	1	37
$v_N = .33$					15	1	−17	1
$v_O = 1.74$					3	89	3	−96
ΣY	0	−1	0	0	0	1	−3	0

From the total deflections v_{tot} the direct stress in the segments of the longitudinals can be calculated with the aid of equation 7. The calculations are presented in the table to follow. It should be noted that $E/L_{mn} = 30 \times 10^6/8 = 3.75 \times 10^6$ pounds per square inch per inch for every segment of longitudinal.

Direct Stress in Stringers

Member mn	v_m tot	v_n tot	(v_n tot−v_m tot)	Stress (psi)
AE, DH	−4.91	−2.75	2.16	810
EJ, HM	−2.75	−1.06	1.69	634
JN, MQ	−1.06	.33	1.39	521
BF, CG	−.27	0	.27	101
FK, GL	0	.72	.72	270
KO, LP	.72	1.74	1.02	382

The values of the stresses calculated with the successive approximation method are compared in figure 23 with the experimental values obtained for the model. The experimental curves are those of direct stress distribution in the stringers for the 240 pound load increment. Since the stress values calculated by the successive approximation method are assumed uniform along the stringer over each panel, they give constant stress lines for each stringer segment. The experimental and calculated stress distribution curves show reasonably good agreement.

As the model contained large unsupported panels of flat sheet, the shear rigidity may have been smaller than calculated theoretically. Therefore the successive approximations procedure was repeated assuming the shear rigidity one-quarter its formerly used value. These calculations are presented in tables 1 to 5, and the resulting direct stress in stringers is shown by the dotted lines in figure 23. If the stress curves so obtained are compared with the experimental curves, closer agreement than that formerly obtained can be seen for the central stringers. It should also be noticed that the new stress values are very near the first ones obtained, notwithstanding the fact that the shear rigidity was assumed to be much different. This indicates that large errors in the assumption of the shear rigidity of the sheet cause but small differences in the final results.

Calculations by Successive Approximations for the Curved Sheet Model

The calculations for the curved model were slightly different from those for the flat model since the purpose of these calculations was not to present an example of the method but to check its accuracy against measured values of the stresses in run J (3000-lb load condition). A sketch of the developed model, showing the external loads and identifying the joints, is given in figure 24.

The influence coefficients and the first rows of the operations table (tables 6 and 7) were calculated as outlined earlier in this section except for the consideration of the shear: in the unit problem illustrated by figure 19 the total shear reaction was assumed to act at the moving point (point C). In the tables pertaining to the curved model minus signs are omitted and negative numbers are underlined.

Since the loading was only approximately symmetrical, it was necessary to balance all 25 joints individually. Advantage was taken of a particular group displacement in order to reduce the labor of relaxation. This group displacement consisted of a simultaneous displacement of all joints to the positions derived for the deflected shape of the model in the Analysis of Test Results. The forces corresponding to these displacements are shown in the second rows of the operations table (table 7).

The original external loads and the effect of the group displacement are shown in table 8. It may be seen from the last row of this table that the original forces were greatly reduced by this displacement.

It was observed that, although there were comparatively large unbalanced forces present at the individual joints, the algebraic sum of the unbalanced forces along any one stringer was not excessive. Trial calculations proved that unbalanced forces of this kind can best be reduced by displacing individual points of the stringer relative to one of the points which is held fixed. The total load on the stringer is not greatly affected by such displacements, since only by relative displacements of adjacent stringers can it be materially changed.

The manner in which this scheme was employed to expedite the convergence of the relaxation may be seen from an examination of the relaxation table (table 9). It will be observed that the central point of the stringer was chosen as the fixed point.

A departure was made from the practice of the preceding example in that unchanged residual forces were not rewritten at each step of the relaxation procedure. An effort was made first to reduce the unbalanced forces on the central stringer (C, H, N, S, X), since the greatest individual unbalanced force occurred along this stringer as may be seen in the first line of the relaxation table. The algebraic sum of the forces on the central stringer was 112 pounds. If this force had been divided among the joints in a manner proportionate to the forces resulting from a block displacement of the stringer (shown in the operations table, table 7), there would have been 14 pounds at each end point and 28 pounds at each inner point. Since the sum of the forces at S and X equaled -344 pounds and the desired sum of the forces at these points was 42 pounds, it was necessary to add 386 pounds to the lower part of the stringer by taking this force away from point N through a displacement of point S.

Next a displacement of X was taken to eliminate the force at X and to reduce the force at S. After completion of these steps of the procedure the sum of the forces at S and X was found to be greater than the 42 pounds desired. The difference was due to the shear forces introduced by the large displacements necessary to balance the large force originally at X.

The same system of relaxations was employed to reduce the forces at H and C. In all the relaxations so far performed, the central point was not displaced.

The total unbalanced force on the central stringer was then found to be 280 pounds, which could be divided into forces of 35 pounds at the end points and 70 pounds at the inner points. Since the total force at S and X was 256 pounds and 105 pounds was desired, point S was displaced to add 154 pounds to point N. Next point X was displaced to balance roughly the resultant forces at S and X.

The procedure was continued until there were reasonably small positive forces left at all the joints along the stringer. Then a block displacement of the stringer was taken to transfer the loads to the adjacent stringers. After the block displacement a few local adjustments served to reduce the maximum unbalanced force along the central stringer to less than 50 pounds at any joint.

The unbalances on stringers A, F, L, Q, V, and B, G, M, R, W were reduced by following the same general procedure, that is, leaving the central points L and M in their original positions. Then a block displacement of stringer B, G, M, R, W, followed by small local displacements, reduced the maximum unbalanced force to 43 pounds.

The goal had arbitrarily been set at 50 pounds maximum residual force, which corresponded to 3-1/2 percent of the maximum external load, but the method could have been continued to reduce this residual force to any desired value.

The check table (table 10) shows that the final residual forces are sufficiently small to make further relaxations unnecessary. The direct stresses in the stringers were calculated by the same procedure as was employed with the flat model (table 11). The shear stress distribution was determined with the aid of equation (8). These calculations are contained in table 12.

A comparison of experimental and calculated values of direct stress in stringers and of shear stress in sheet is shown in figures 25 and 26. It may be seen that good qualitative agreement was obtained. The error was greatest at the point of maximum stress where the concentrated load was introduced. It is believed that the assumption of a constant effective width of sheet throughout the model was largely responsible for this deviation. In stringers I, II, and IV there appears to be a systematic deviation between experimental and calculated values. This observation, however, is not necessarily correct since the stress in the stringers was measured only at sections B and D, and the experimental curves were drawn in the simplest possible way between these points.

CONCLUSIONS

The convergence of the successive approximation procedure is rapid in the calculation of the stresses in a flat or cylindrical reinforced sheet with concentrated axial loads applied to the end points of the longitudinal reinforcements, provided the end points of the longitudinals are not restrained from axial displacement, if the succession of steps listed below is followed in the balancing procedure:

(1) Displace individual points of one longitudinal until the unbalanced forces along it attain magnitudes approximately proportional to the forces caused by a block displacement of the entire longidinal.

(2) Displace the longitudinal as a block by an amount sufficient to balance as much as possible of the unbalanced forces remaining after the steps described under (1) are performed.

(3) Repeat the steps described under (1) and (2) with the same longitudinal and the others contained in the structure until the unbalanced forces attain values which can be considered negligibly small.

The effective width of sheet is not constant along the longitudinals. In the experiments carried out the ratio of effective width to total width was found to vary from 0.33 to 0.72 in the curved specimen, from 0.21 to 0.32 in the flat specimen. Nevertheless, reasonably good agreement was obtained between stresses measured in experiment and those calculated on the assumption of a constant average value of the effective width.

In the calculations by successive approximations a reduction of the value of the shear modulus to one-quarter its theoretical value did not cause any material changes in the stresses computed.

Polytechnic Institute of Brooklyn,
Brooklyn, New York, February, 1944.

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TABLE 1.- INFLUENCE COEFFICIENTS FOR FLAT MODEL

WITH REDUCED SHEAR RIGIDITY

[lb/in. x 10^{-4}]

nm	AB	AE	AF	BE	BF	EF	EJ	EK
$\widehat{yy}_{nm}$	0.50	48.3	0.50	0.50	52.7	1.00	48.3	0.50

nm	FJ	FK	JK	JN	JO	KN	KO	NO
$\widehat{yy}_{nm}$	0.50	52.7	1.00	48.3	0.50	0.50	52.7	0.50

TABLE 2.- OPERATION TABLE FOR FLAT MODEL WITH REDUCED SHEAR RIGIDITY

[Forces in lb, displacements in in. x 10^4]

Displ.	Y_A	Y_B	Y_E	Y_F	Y_J	Y_K	Y_N	Y_O
$v_A = 1$	-49.3	.50	48.3	.50				
$v_B = 1$	.50	-53.7	.50	52.7				
$v_E = 1$	48.3	.50	-98.6	1.00	48.3	.50		
$v_F = 1$	.50	52.7	1.00	-107.4	.50	52.7		
$v_J = 1$			48.3	.50	-98.6	1.00	48.3	.50
$v_K = 1$			.50	52.7	1.00	-107.4	.50	52.7
$v_N = 1$					48.3	.50	-49.3	.50
$v_O = 1$					.50	52.7	.50	-53.7
$v_{block} = 1$	-1.00	1.00	-2.00	2.00	-2.00	2.00	-1.00	1.00

NOTE: v_{block} corresponds to $v_A = v_E = v_J = v_N = 1$ simultaneously.

TABLE 3.- RELAXATION TABLE FOR FLAT MODEL

WITH REDUCED SHEAR RIGIDITY

	Y_A	Y_B	Y_E	Y_F	Y_J	Y_K	Y_N	Y_O
External loads	-120						60	60
v_A = -2.23	110	-1	-108	-1				
v_J = 1.82	-10	-1	-108 88	-1 1	-179	2	60 88	60 1
v_N = 3.29	-10	-1	-20	0	-179 159	2 2	148 -162	61 2
v_B = -.20	-10	-1 11	-20	0 -11	-20	4	-14	63
v_K = .59	-10	10	-20	-11 31	-20 1	4 -63	-14	63 31
v_O = 1.50	-10	10	-20	20	-19 1	-59 79	-14 1	94 -81
v_{block} = -10.0	-10 10	10 -10	-20 20	20 -20	-18 20	20 -20	-13 10	13 -10
	0	0	0	0	2	0	-3	3

TABLE 4.- CHECK TABLE FOR FLAT MODEL

WITH REDUCED SHEAR RIGIDITY

v_{tot}	Y_A	Y_B	Y_E	Y_F	Y_J	Y_K	Y_N	Y_O
External loads	-120						60	60
v_A = -12.23	603	-6	-591	-6				
v_B = -.20		11		-11				
v_E = -10.00	-483	-5	986	-10	-483	-5		
v_F = 0								
v_J = -8.18			-395	-4	807	-8	-395	-4
v_K = .59				31	1	-63		31
v_N = -6.71					-324	-3	331	-3
v_O = 1.50					1	79	1	-81
ΣY	0	0	0	0	2	0	-3	3

TABLE 5.- DIRECT STRESS IN STRINGERS FOR FLAT MODEL

WITH REDUCED SHEAR RIGIDITY

Member mn	v_m tot	v_n tot	(v_n tot-v_m tot)	Stress (psi)
AE, DH	-12.23	-10.0	2.23	836
EJ, HM	-10.0	-8.18	1.82	683
JN, MQ	-8.18	-6.71	1.47	551
BF, CG	-.20	0	.20	75
FK, GL	0	.59	.59	221
KO, LP	.59	1.5	.91	341

TABLE 6. INFLUENCE COEFFICIENTS FOR CURVED MODEL

Point	$\widehat{YY}$ x 10^{-4}	Point	$\widehat{YY}$ x 10^{-4}	Point	$\widehat{YY}$ x 10^{-4}
A-B	2.62	H-J	5.25	O-U	2.62
A-F	70.0	H-M	2.62	P-T	2.62
A-G	2.62	H-N	77.5	P-U	70.0
B-C	2.62	H-O	2.62	Q-V	70.0
B-F	2.62	J-K	5.25	Q-W	2.62
B-H	2.62	J-O	77.5	R-S	5.25
B-G	77.5	J-N	2.62	R-V	2.62
C-D	2.62	J-P	2.62	R-W	77.5
C-G	2.62	K-O	2.62	R-X	2.62
C-H	77.5	K-P	70.0	S-T	5.25
C-J	2.62	L-M	5.25	Q-R	5.25
D-E	2.62	L-Q	70.0	S-W	2.62
D-H	2.62	L-R	2.62	S-X	77.5
D-J	77.5	M-N	5.25	S-Y	2.62
D-K	2.62	M-Q	2.62	T-U	5.25
E-J	2.62	M-R	77.5	T-X	2.62
E-K	70.0	M-S	2.62	T-Y	77.5
F-G	5.25	N-O	5.25	T-Z	2.62
F-L	70.0	N-R	2.62	U-Y	2.62
F-M	2.62	N-S	77.5	U-Z	70.0
G-H	5.25	N-T	2.62	V-W	2.62
G-L	2.62	O-P	5.25	W-X	2.62
G-M	77.5	O-S	2.62	X-Y	2.62
G-N	2.62	O-T	77.5	Y-Z	2.62

TABLE 7. OPERATIONS TABLE FOR CURVED MODEL. SHEET 1.

Operation $\times 10^4$	Y_A	Y_B	Y_C	Y_D	Y_E	Y_F	Y_G	Y_H	Y_J	Y_K	Y_L	Y_M	Y_N	Y_O	Y_P	Y_Q	Y_R	Y_S	Y_T	Y_U	Y_V	Y_W	Y_X	Y_Y	Y_Z
$V_A=1$ $V_A=8.9$	75.25 670	2.62 23				70.0 623	2.62 23																		
$V_B=1$ $V_B=4.45$	2.62 11	88.0 365	2.62 11			2.62 11	77.5 322	2.62 11																	
$V_C=1$ $V_C=0$		2.62 0	88.0 0	2.62 0			2.62 0	77.5 0	2.62 0																
$V_D=1$ $V_D=36$			2.62 9	88.0 317	2.62 9			2.62 9	77.5 279	2.62 9															
$V_E=1$ $V_E=7.55$				2.62 20	75.25 568				2.62 20	70.0 529															
$V_F=1$ $V_F=3.6$	70.0 252	2.62 9				150.5 541	5.25 19				70.0 252	2.62 9													
$V_G=1$ $V_G=.20$	2.62 .5	77.5 16	2.62 .5			5.25 1.1	176.0 35	5.25 1.1			2.62 .5	77.5 16	2.62 .5												
$V_H=1$ $V_H=3.8$		2.62 10	77.5 294	2.62 10			5.25 20	176.0 609	5.25 20			2.62 10	77.5 294	2.62 10											
$V_J=1$ $V_J=.6$			2.62 1.5	77.5 4.65	2.62 1.5			5.25 3	176 105.5	5.25 3			2.62 1.5	77.5 46.5	2.62 1.5										
$V_K=1$ $V_K=4.35$				2.62 11.5	70.0 304				5.25 23	150.5 655				2.62 11.5	70.0 304										

TABLE 7. OPERATIONS TABLE FOR CURVED MODEL. SHEET 2.

Operation $\times 10^4$	Y_A	Y_B	Y_C	Y_D	Y_E	Y_F	Y_G	Y_H	Y_J	Y_K	Y_L	Y_M	Y_N	Y_O	Y_P	Y_Q	Y_R	Y_S	Y_T	Y_U	Y_V	Y_W	Y_X	Y_Y	Y_Z
$V_L = 1$ $V_L = 3.7$						70.0 259	2.62 10				150.5 556	5.25 19.5				70.0 259	2.62 10								
$V_M = 1$ $V_M = 3.0$						2.62 8.0	77.5 232	2.62 8.0			5.25 16.0	176.0 528	5.25 16.0			2.62 8.0	77.5 232	2.62 8.0							
$V_N = 1$ $V_N = 10.6$							2.62 28	77.5 821	2.62 28			5.25 56	176 1870	5.25 56			2.62 28	77.5 821	2.62 28						
$V_O = 1$ $V_O = 3.3$								2.62 9.0	77.5 256	2.62 9.0			5.25 17	176.0 580	5.25 17			2.62 9.0	77.5 256	2.62 9.0					
$V_P = 1$ $V_P = 2.55$									2.62 7.0	70.0 178				5.25 13	150.5 384				2.62 7.0	70.0 178					
$V_Q = 1$ $V_Q = 2.7$											70.0 189	2.62 7.0				150.5 406	5.25 14				70.0 189	2.62 7.0			
$V_R = 1$ $V_R = 4.65$											2.62 12	77.5 360	2.62 12			5.25 24	176 819	5.25 24			2.62 12	77.5 360	2.62 12		
$V_S = 1$ $V_S = 23.4$												2.62 61	77.5 1815	2.62 61			5.25 123	176 4120	5.25 123			2.62 61	77.5 1815	2.62 61	
$V_T = 1$ $V_T = 4.7$													2.62 12	77.5 364	2.62 12			5.25 25	176 826	5.25 25			2.62 12	77.5 364	2.62 12
$V_U = 1$ $V_U = 1.65$														2.62 4	70.0 115				5.25 9	150.5 248				2.62 4	70.0 115

TABLE 7. OPERATIONS TABLE FOR CURVED MODEL. SHEET 3.

Operation $\times 10^4$	Y_A	Y_B	Y_C	Y_D	Y_E	Y_F	Y_G	Y_H	Y_J	Y_K	Y_L	Y_M	Y_N	Y_O	Y_P	Y_Q	Y_R	Y_S	Y_T	Y_U	Y_V	Y_W	Y_X	Y_Y	Y_Z
$V_V = 1$ $V_V = 2.15$																70.0 171	2.62 6				75.25 184	2.62 6			
$V_W = 1$ $V_W = 3.15$																2.62 13	77.5 399	2.62 13			2.62 13	88.0 453	2.62 13		
$V_X = 1$ $V_X = 4.30$																	2.62 115	77.5 3390	2.62 115			2.62 115	88.0 3860	2.62 115	
$V_Y = 1$ $V_Y = 5.2$																		2.62 14	77.5 403	2.62 14			2.62 14	88.0 458	2.62 14
$V_Z = 1$ $V_Z = 1.33$																			2.62 4	70.0 94				2.62 4	75.25 102
$V_C = V_H = V_N = V_S = V_X = 1$		5.25	10.5	5.25			10.5	21	10.5			10.5	21	10.5			10.5	21	10.5			5.25	10.5	5.25	
$V_D = V_J = V_O = V_T = V_Y = 1$			5.25	10.5	5.25			10.5	21	10.5			10.5	21	10.5			10.5	21	10.5			5.25	10.5	5.25
$V_B = V_G = V_M = V_R = V_W = 1$	5.25	10.5	5.25			10.5	21	10.5			10.5	21	10.5			10.5	21	10.5			5.25	10.5	5.25		

TABLE 7. OPERATIONS TABLE FOR CURVED MODEL. SHEET 3.

Operation $\times 10^4$	Y_A	Y_B	Y_C	Y_D	Y_E	Y_F	Y_G	Y_H	Y_J	Y_K	Y_L	Y_M	Y_N	Y_O	Y_P	Y_Q	Y_R	Y_S	Y_T	Y_U	Y_V	Y_W	Y_X	Y_Y	Y_Z
$V_V=1$ $V_V=2.45$																70.0 171	2.62 6				75.25 184	2.62 6			
$V_W=1$ $V_W=5.15$																2.62 13	77.5 399	2.62 13			2.62 15	88.0 453	2.62 15		
$V_X=1$ $V_X=43$[illegible]																	2.62 115	77.5 3390	2.62 115			2.62 115	88.0 3860	2.62 115	
$V_Y=1$ $V_Y=5.2$																		2.62 14	77.5 403	2.62 14			2.62 14	88.0 458	2.62 14
$V_Z=1$ $V_Z=1.33$																			2.62 4	70.0 94				2.62 4	75.25 102
$V_C=V_H=V_N=V_S=V_X=1$		5.25	10.5	5.25			10.5	21	10.5			10.5	21	10.5			10.5	21	10.5			5.25	10.5	5.25	
$V_D=V_J=V_O=V_T=V_Y=1$			5.25	10.5	5.25			10.5	21	10.5			10.5	21	10.5			10.5	21	10.5			5.25	10.5	5.25
$V_B=V_G=V_M=V_R=V_W=1$	5.25	10.5	5.25			10.5	21	10.5			10.5	21	10.5			10.5	21	10.5			5.25	10.5	5.25		

TABLE 8. GROUP RELAXATION FOR CURVED MODEL.

JOINTS	A	B	C	D	E	F	G	H	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
ORIGINAL LOADS	272	325	271	307	256	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1466	0	0
GROUP RELAXATION	408	359	276	342	257	343	129	153	131	49	144	61	298	71	5	21	48	184	79	24	20	70	1994	74	13
RESIDUAL	136	34	5	35	1	343	129	153	131	49	144	61	298	71	5	21	48	184	79	24	20	70	528	74	13

TABLE 9. RELAXATION TABLE FOR CURVED MODEL. SHEET 1.

Operation $\times 10^4$	Y_A	Y_B	Y_C	Y_D	Y_E	Y_F	Y_G	Y_H	Y_J	Y_K	Y_L	Y_M	Y_N	Y_O	Y_P	Y_Q	Y_R	Y_S	Y_T	Y_U	Y_V	Y_W	Y_X	Y_Y	Y_Z
Residual	136	34	5	35	1	343	129	153	131	49	144	61	298	71	5	21	48	184	79	24	20	70	528	74	13
$V_S = 4.98$												13	386	13			26	877	26			13	386	13	
												74	88	84			22	1061	53			57	914	61	
$V_X = 10.39$																	27	805	27			27	914	27	
																	5	256	26			30	0	34	
$V_H = 1.50$		4	116	4			8	264	8			4	116	4											
		38	121	39			121	111	123			70	28	80											
$V_C = 1.38$		4	121	4			4	107	4																
		42	0	43			117	4	119																
$V_S = 1.99$												5	154	5			10	350	10			5	154	5	
												65	182	75			5	94	36			35	154	39	
$V_X = 1.73$																	5	134	5			5	152	5	
																	10	40	41			40	2	44	
$V_H = 1.42$		4	110	4			7	250	7			4	110	4											
		38	110	39			124	246	126			69	72	79											
$V_C = 1.71$		4	150	4			4	132	4																
$V_C = V_H = V_M$		34	40	35			128	114	130																
$= V_S = V_X = 2.5$		13	26	13			26	53	26			26	53	26			26	53	26			13	26	13	
	136	47	14	48	1	343	102	61	104	49	144	43	19	53	5	21	36	13	67	24	20	53	24	57	13

TABLE 9. RELAXATION TABLE FOR CURVED MODEL. SHEET 2.

OPERATION $\times 10^4$	Y_A	Y_B	Y_C	Y_D	Y_E	Y_F	Y_G	Y_H	Y_J	Y_K	Y_L	Y_M	Y_N	Y_O	Y_P	Y_Q	Y_R	Y_S	Y_T	Y_U	Y_V	Y_W	Y_X	Y_Y	Y_Z
RESIDUAL	136	47	14	48	1	343	102	61	104	49	144	43	19	53	5	21	36	13	67	24	20	53	24	57	13
$V_N = .53$							1	41	1			3	93	3			1	41	1						
							103	20	105			46	112	56			35	54	66						
$V_S = 1.19$												3	92	3			6	209	6			3	92	3	
												49	20	59			29	155	60			50	116	54	
$V_X = 1.83$																	5	142	5			5	161	5	
	136	47	14	48	1	343	103	20	105	49	144	49	20	59	5	21	24	13	55	24	20	45	45	49	13
$V_F = 2.96$	207	8				445	16				207	8													
	71	39				102	119				63	57													
$V_A = 1.21$	91	3				85	3																		
	20	36				17	122																		
$V_Q = .59$											41	2				89	3				41	2			
											22	55				68	27				61	47			
$V_V = .93$																65	2				70	2			
																3	29				9	49			
$V_G = 1.11$	3	86	3			6	195	6			3	86	3												
	17	50	11			11	73	14			25	141	17												
$V_B = .74$	2	65	2			2	57	2																	
	15	15	9	48	1	9	16	12	105	49	25	141	17	59	5	3	29	13	55	24	9	49	45	49	13

TABLE 9. RELAXATION TABLE FOR CURVED MODEL. SHEET 3.

OPERATION ×10⁴	Y_A	Y_B	Y_C	Y_D	Y_E	Y_F	Y_G	Y_H	Y_J	Y_K	Y_L	Y_M	Y_N	Y_O	Y_P	Y_Q	Y_R	Y_S	Y_T	Y_U	Y_V	Y_W	Y_X	Y_Y	Y_Z
RESIDUAL	15	15	9	48	1	9	16	12	105	49	25	141	17	59	5	3	29	13	55	24	9	49	45	49	13
V_R = 1.01											3	78	3			5	178	5			3	78	3		
											22	63	20			2	149	18			6	127	48		
V_W = 1.65																4	128	4			4	145	4		
V_B = V_G = V_M =																6	21	22			2	18	52		
V_R = V_W = 1.00	5	11	5			11	21	11			11	21	11			11	21	11			5	11	5		
	10	26	4			2	37	1			33	42	9			5	0	11			7	7	47		
V_J = .74			2	57	2			4	130	4			2	57	2										
			2	9	1			3	25	53			7	116	7										
V_T = 1.34													4	104	4			7	236	7			4	104	4
													11	12	3			18	181	31			51	153	17
V_Y = 2.00																		5	155	5			5	176	5
	10	26	2	9	1	2	37	3	25	53	33	42	11	12	3	5	0	23	26	36	7	7	56	23	22
V_E = .33				1	25				1	23															
				8	26				26	30															
V_X = .15																	0	12	0			0	13	0	
RESIDUAL	10	26	2	8	26	2	37	3	26	30	33	42	11	12	3	5	0	35	26	36	7	7	43	23	22

TABLE 10. CHECK TABLE FOR CURVED MODEL.

OPERA-TION $\times 10^4$	Y_A	Y_B	Y_C	Y_D	Y_E	Y_F	Y_G	Y_H	Y_J	Y_K	Y_L	Y_M	Y_N	Y_O	Y_P	Y_Q	Y_R	Y_S	Y_T	Y_U	Y_V	Y_W	Y_X	Y_Y	Y_Z
LOADS	272	325	271	307	256																		1466		
$V_A = 10.11$	761	26				708	26																		
$V_F = 6.56$	460	17				988	34				460	17													
$V_L = 3.70$						259	10				556	19				259	10								
$V_Q = 2.11$											148	6				318	11				148	6			
$V_V = 1.52$																106	4				114	4			
$V_B = 5.89$	15	518	15			15	456	15																	
$V_G = 1.91$	5	148	5			10	336	10			5	148	5												
$V_M = 2.00$						5	155	5			11	352	11			5	155	5							
$V_R = 4.66$											12	361	12			24	820	24			12	361	12		
$V_W = 5.80$																15	450	15			15	510	15		
$V_C = 2.17$		6	191	6			6	168	6																
$V_H = 6.38$		17	494	17			34	1123	34			17	494	17											
$V_N = 12.57$							33	974	33			66	2212	66			33	974	33						
$V_S = 21.72$												57	1683	57			114	3822	114			57	1683	57	
$V_X = 35.96$																	94	2787	94			94	3164	94	
$V_D = 3.60$			9	317	9			9	279	9															
$V_J = .14$				11				1	25	1				11											
$V_O = 3.30$								9	256	9			17	581	17			9	256	9					
$V_T = 6.04$													16	468	16			32	1063	32			16	468	16
$V_Y = 7.20$																		19	558	19			19	634	19
$V_E = 7.22$				19	543				19	505															
$V_K = 4.35$				11	305				23	655				11	305										
$V_P = 2.55$									7	179				13	384				7	179					
$V_U = 1.65$														4	116				9	248				4	116
$V_Z = 1.35$																			4	95				4	102
RESIDUAL	9	25	3	8	27	1	38	2	26	30	34	41	16	12	4	3	1	43	28	34	7	8	47	23	21

TABLE 11.- CALCULATION OF TENSION IN STRINGERS

STRINGER I				STRINGER II			
Symbol	Displacement (in. × 10^{-4})		Stress (psi)	Symbol	Displacement (in. × 10^{-4})		Stress (psi)
v_A	-10.11			v_B	-5.89		
		3.55	1770			3.98	1990
v_F	-6.56			v_G	-1.91		
		2.86	1430			3.91	1950
v_L	-3.70			v_M	2.00		
		1.59	795			2.66	1330
v_Q	-2.11			v_R	4.66		
		.59	295			1.14	570
v_V	-1.52			v_W	5.80		

STRINGER III				STRINGER IV			
v_C	2.17			v_D	-3.60		
		4.21	2110			3.46	1730
v_H	6.38			v_J	-.14		
		6.19	3090			3.44	1720
v_N	12.57			v_O	3.30		
		9.15	4580			2.74	1370
v_S	21.72			v_T	6.04		
		14.24	7120			1.16	580
v_X	35.96			v_Y	7.20		

STRINGER V			
v_E	-7.22		
		2.87	1440
v_K	-4.35		
		1.80	900
v_P	-2.55		
		.90	450
v_U	-1.65		
		.30	150
v_Z	-1.35		

TABLE 12.- CALCULATION OF SHEAR STRESS

STRINGER				
Section	I	II	II – I	Stress
	Displacement (in. × 10^{-4})			(psi)
A	-10.11	-5.89	4.22	739
B	-6.56	-1.91	4.65	815
C	-3.70	2.00	5.70	998
D	-2.11	4.66	6.77	1190
E	-1.52	5.80	7.32	1280
STRINGER				
Section	II	III	III – II	Stress
	Displacement (in. × 10^{-4})			(psi)
A	-5.89	2.17	8.06	1410
B	-1.91	6.38	8.29	1450
C	2.00	12.57	10.57	1850
D	4.66	21.72	17.06	2980
E	5.80	35.96	30.16	5270
STRINGER				
Section	III	IV	IV – III	Stress
	Displacement (in. × 10^{-4})			(psi)
A	2.17	-3.60	-5.77	-1010
B	6.38	-.14	-6.52	-1140
C	12.57	3.30	-9.27	-1620
D	21.72	6.04	-15.68	-2740
E	35.96	7.20	-28.76	-5030
STRINGER				
Section	IV	V	V – IV	Stress
	Displacement (in. × 10^{-4})			(psi)
A	-3.60	-7.22	-3.62	-634
B	-.14	-4.35	-4.21	-736
C	3.30	-2.55	-5.85	-1020
D	6.04	-1.65	-7.69	-1340
E	7.20	-1.35	-8.55	-1500

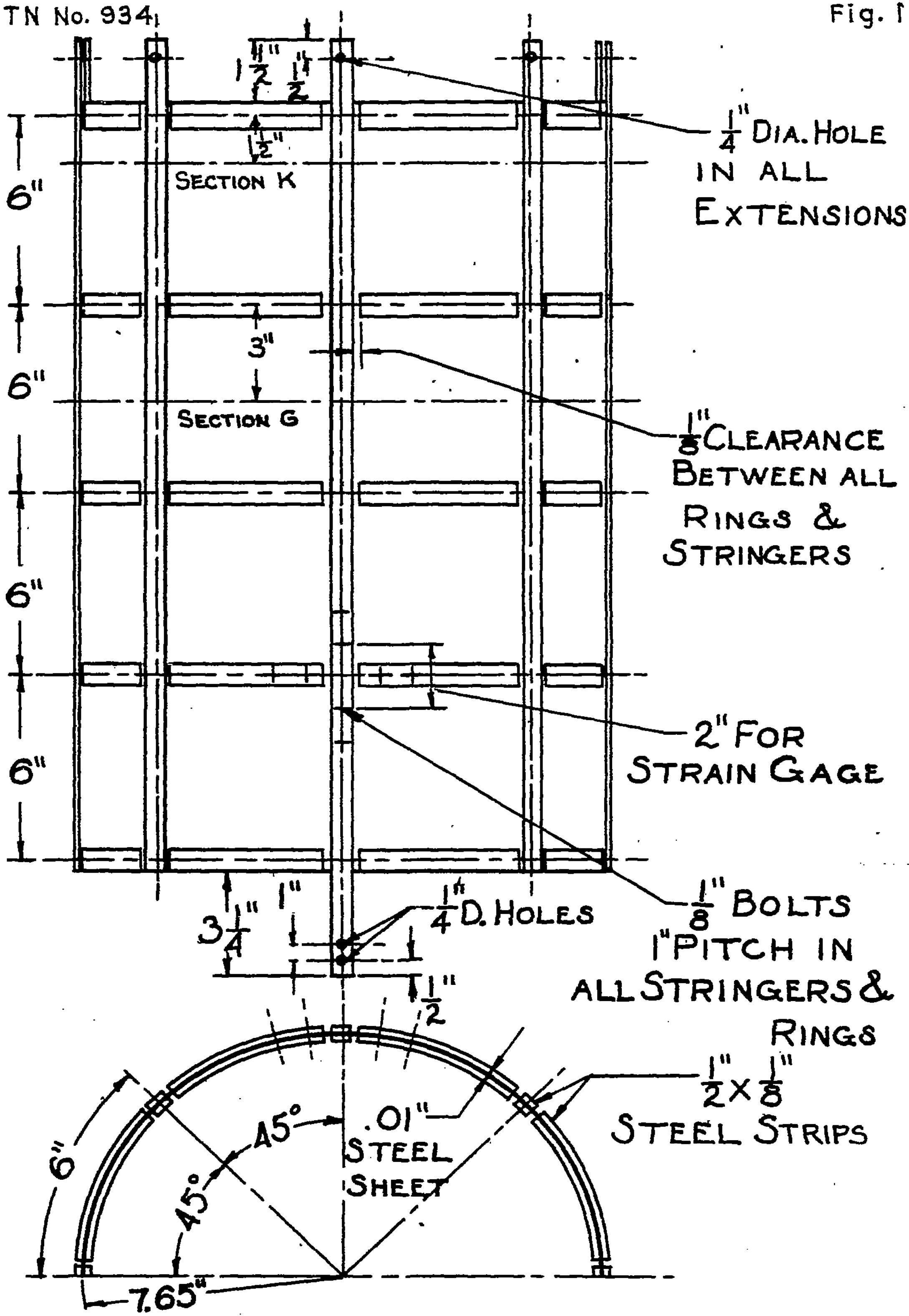


FIG. 1. CURVED TEST MODEL.

Figure 2.- Curved model test set-up.

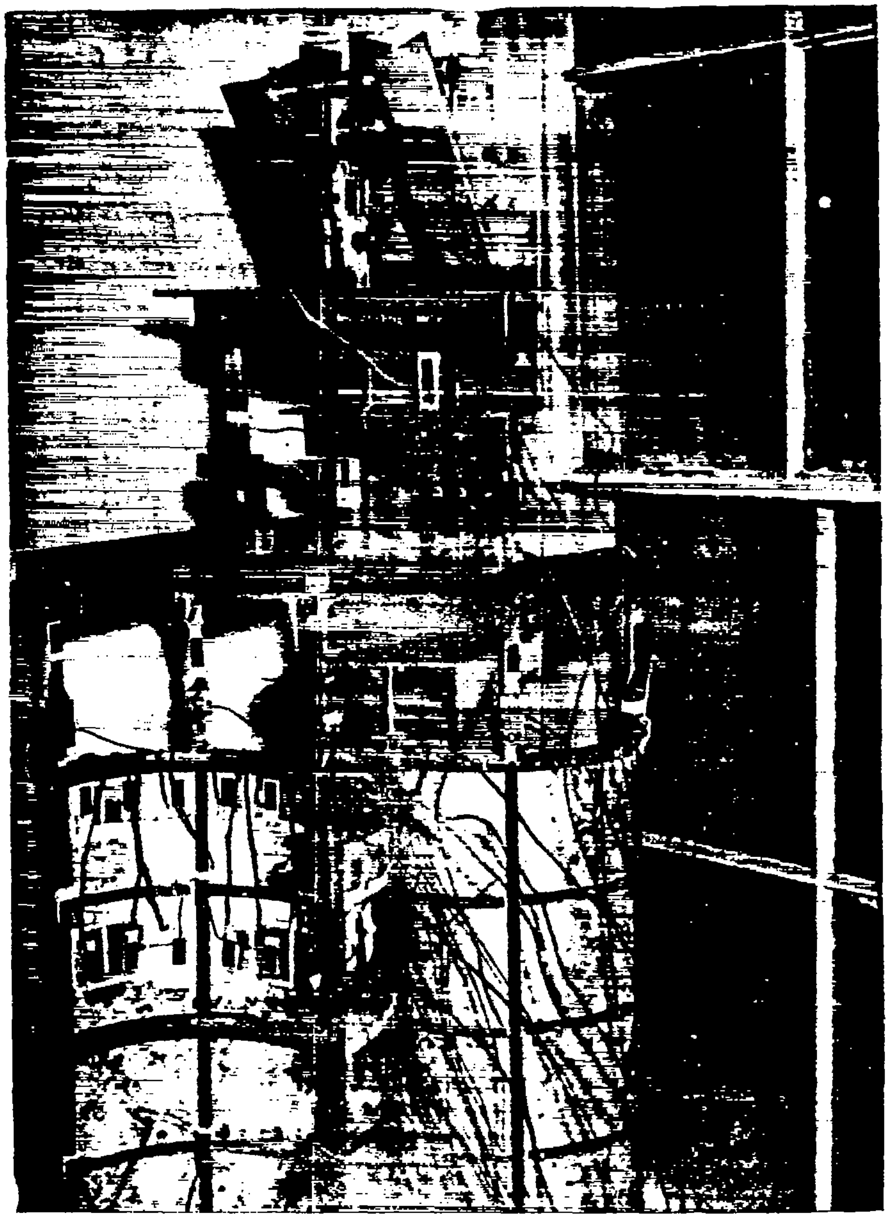


Figure 3.- Upper lever system for curved model.

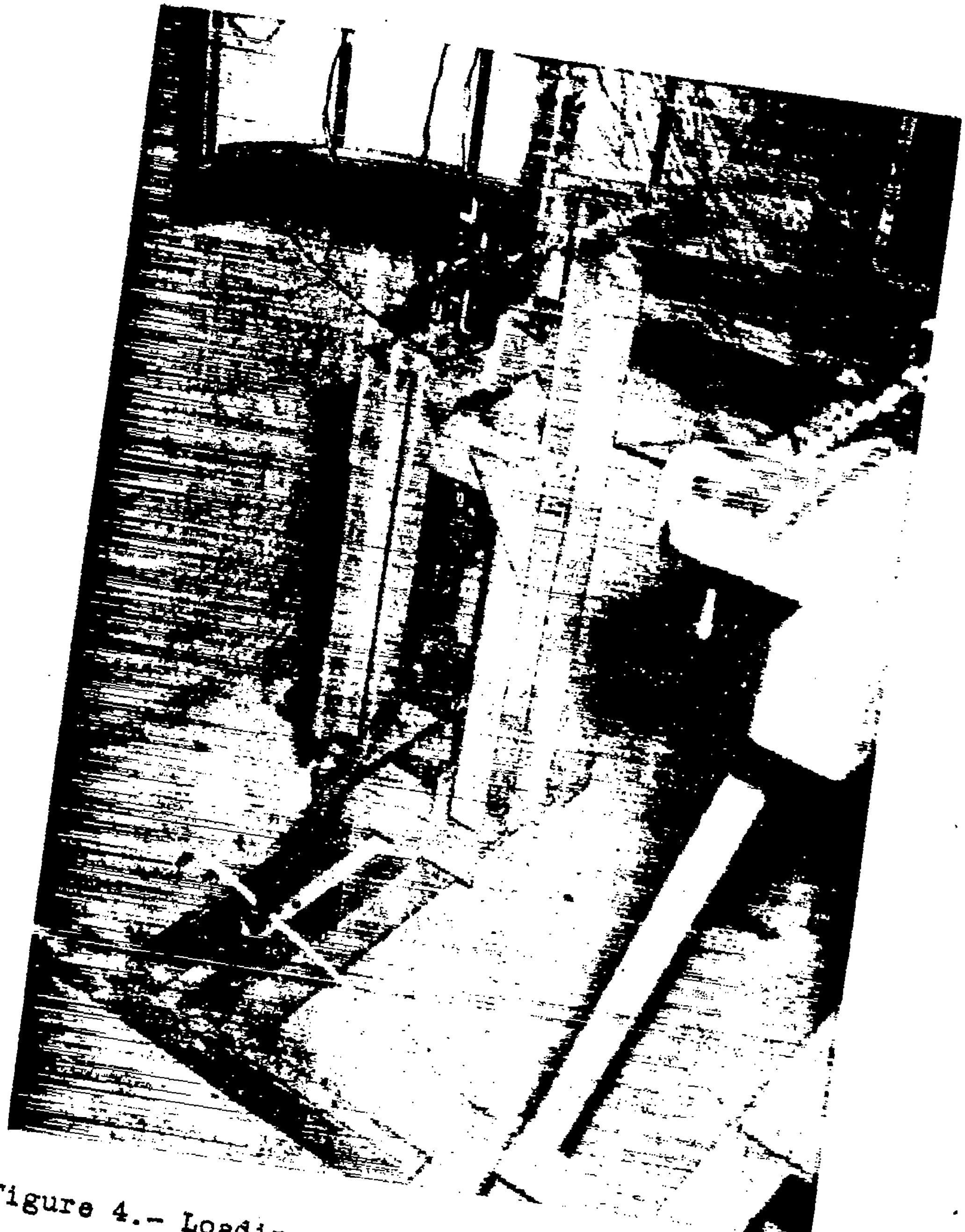


Figure 4.- Loading mechanism for curved model.

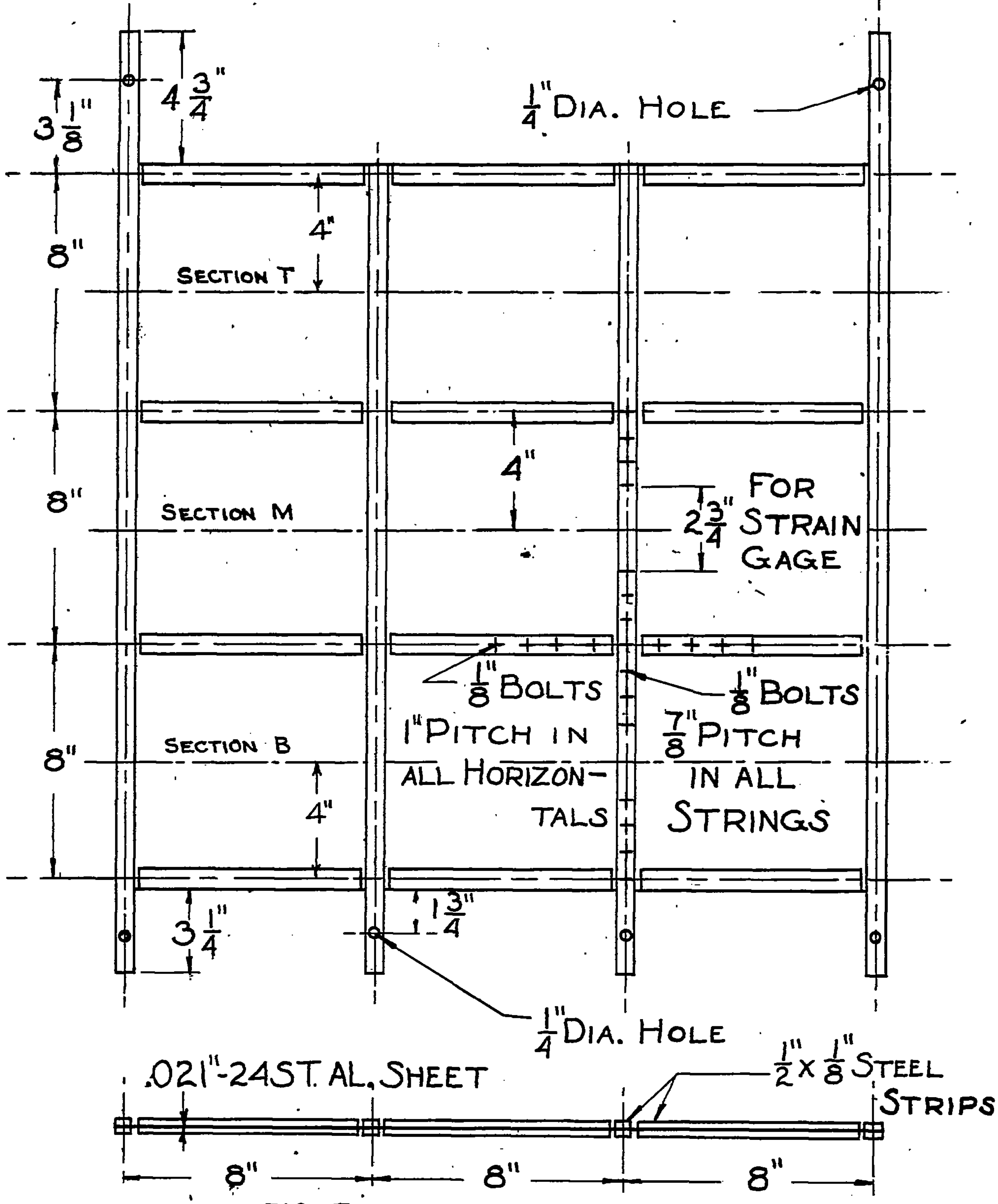


FIG. 5. FLAT TEST MODEL.

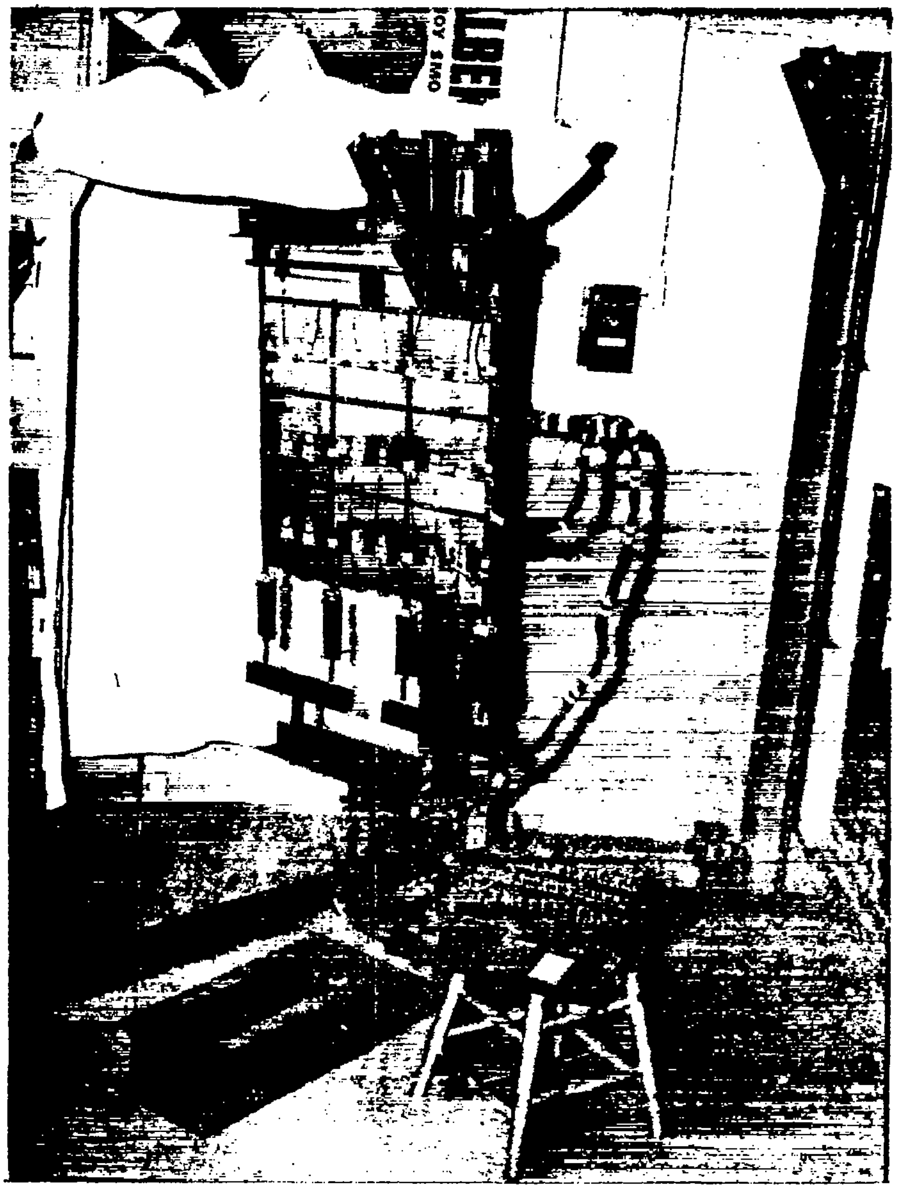


Figure 6.- Flat model test set-up.

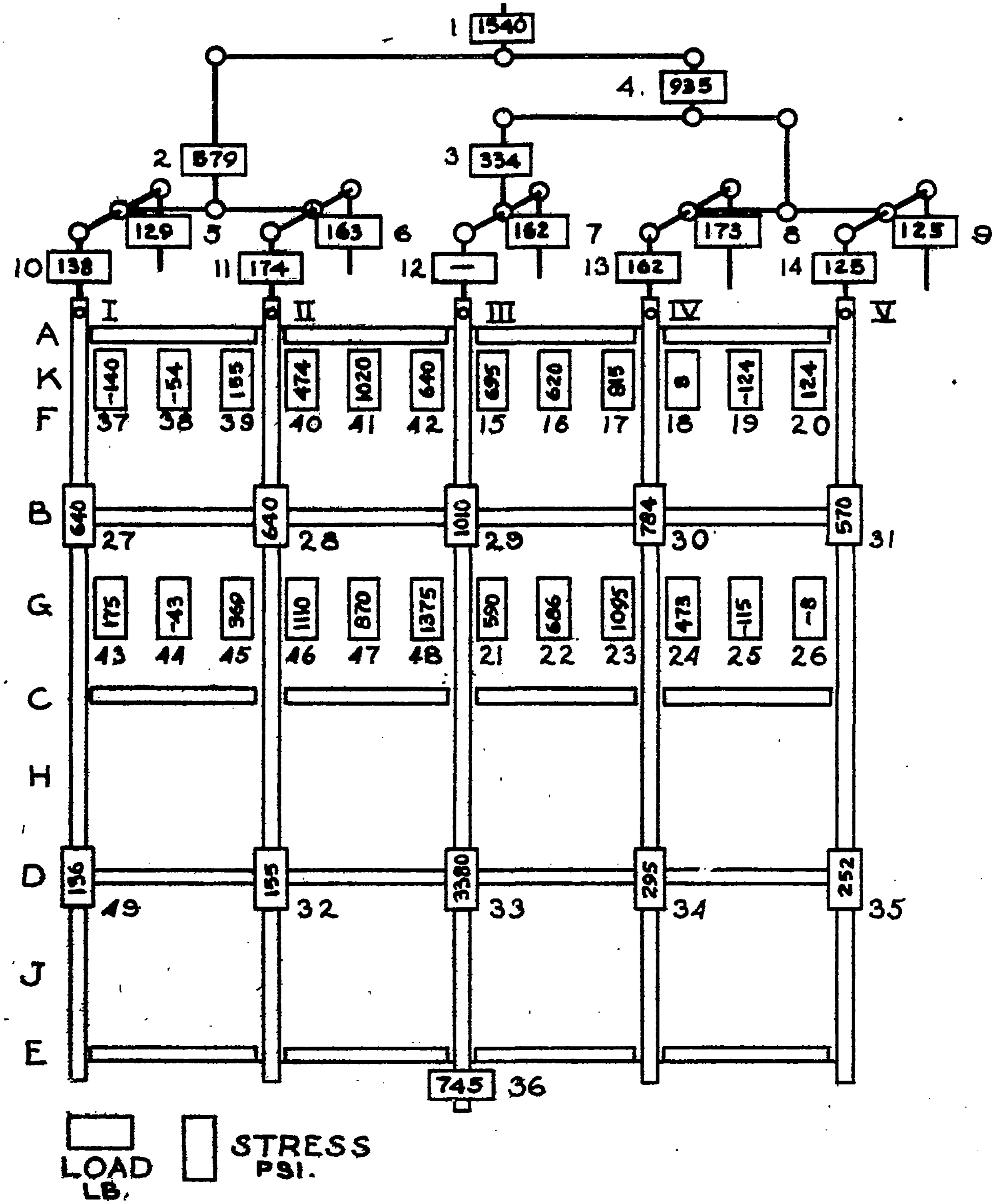


FIG. 7. CURVED MODEL WITH MEASURED LOADS AND STRESSES. 1500 LB. LOAD CONDITION.

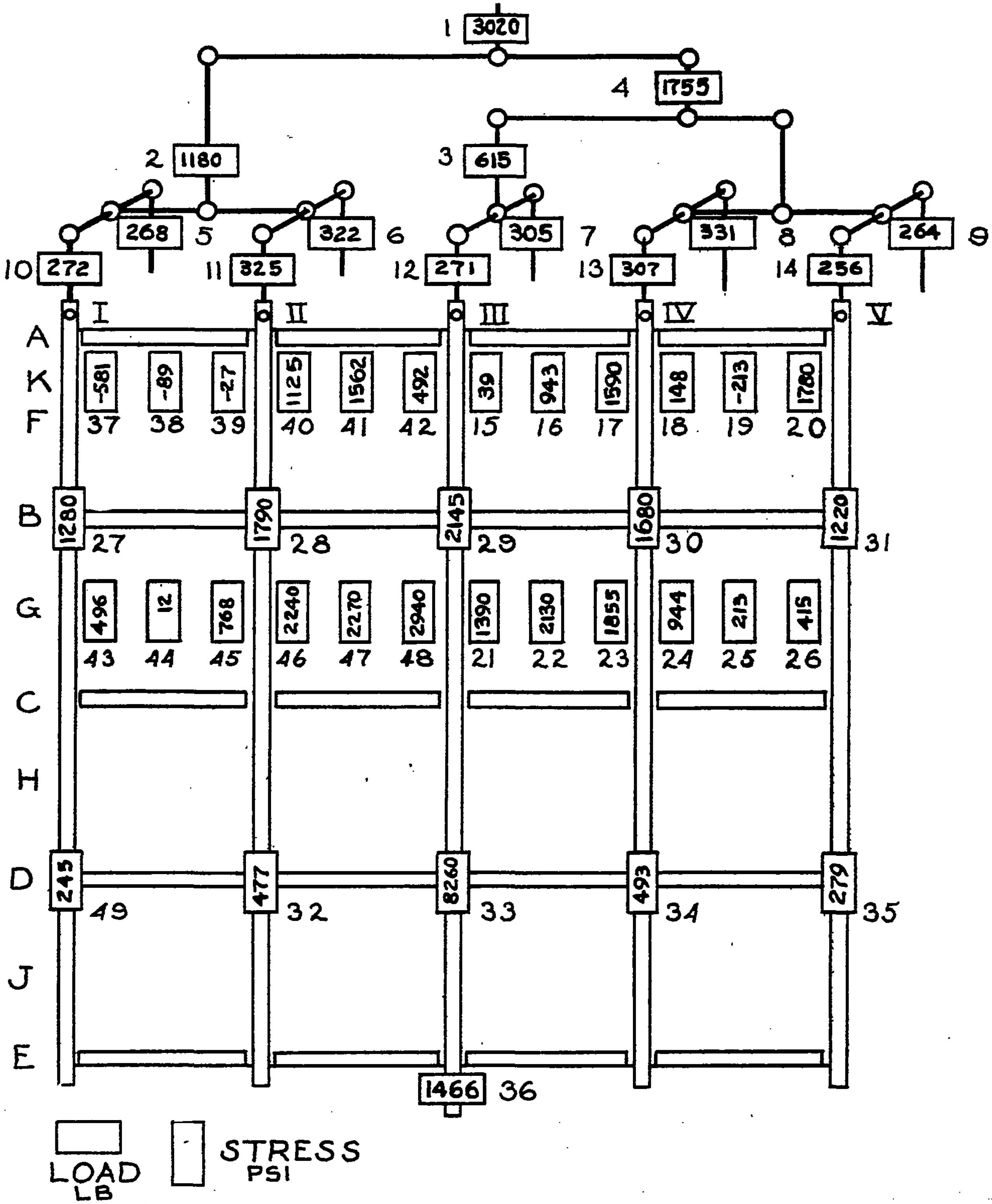


FIG. 8. CURVED MODEL WITH MEASURED LOADS AND STRESSES. 3000 LB LOAD CONDITION.

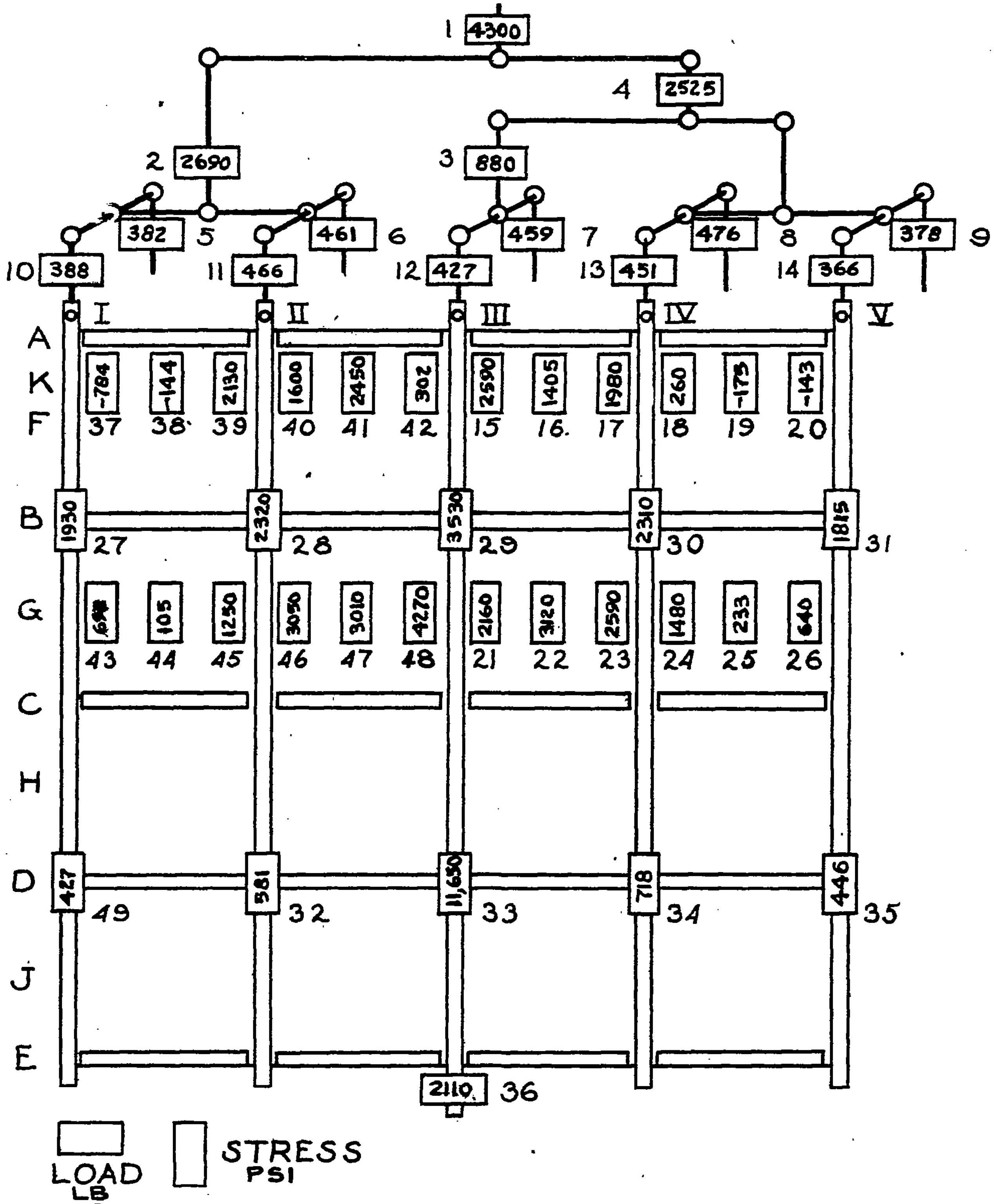


FIG. 9. CURVED MODEL WITH MEASURED LOADS AND STRESSES. 4500 LB LOAD CONDITION.

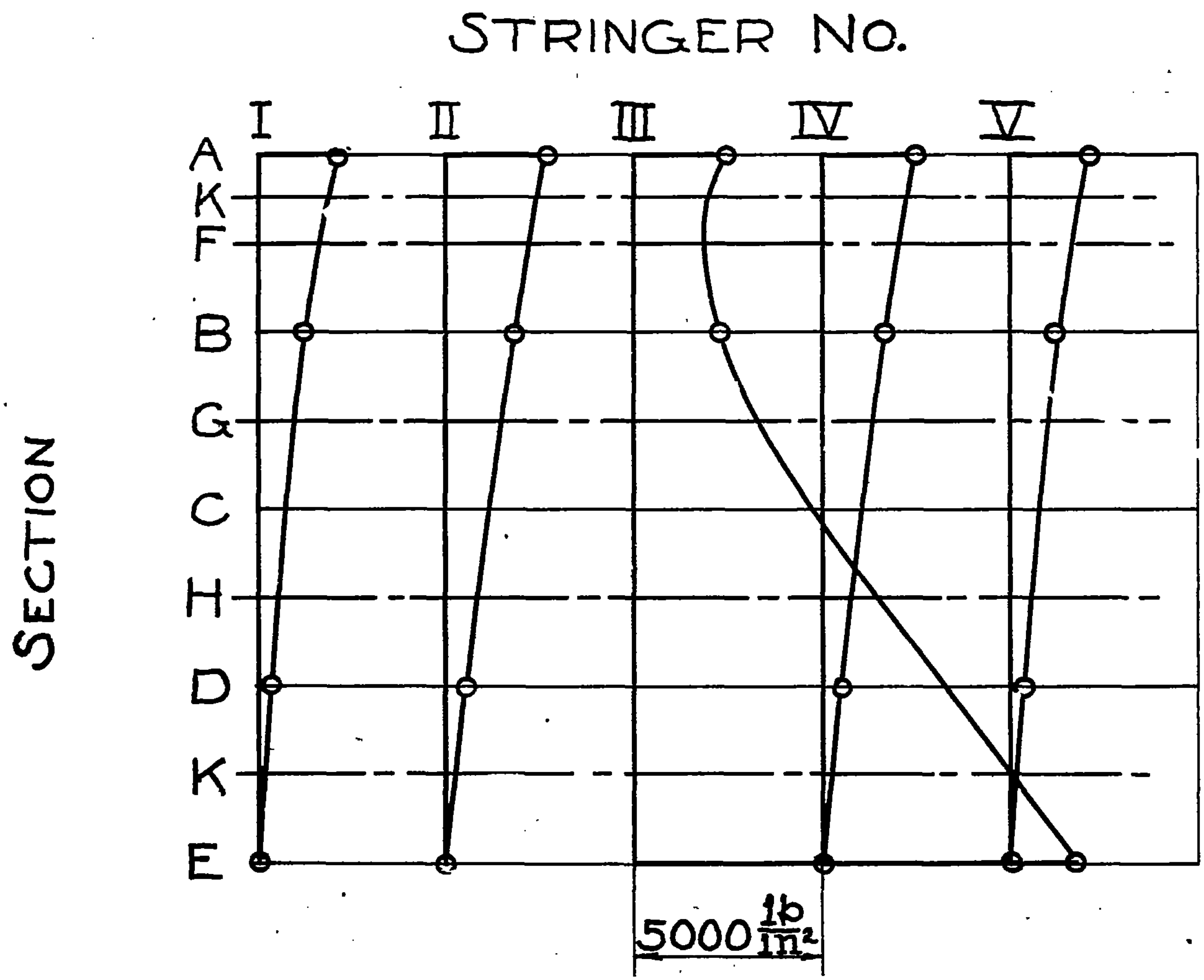


FIG. 10. DIRECT STRESS IN STRINGERS.
3000 LB LOAD CONDITION.

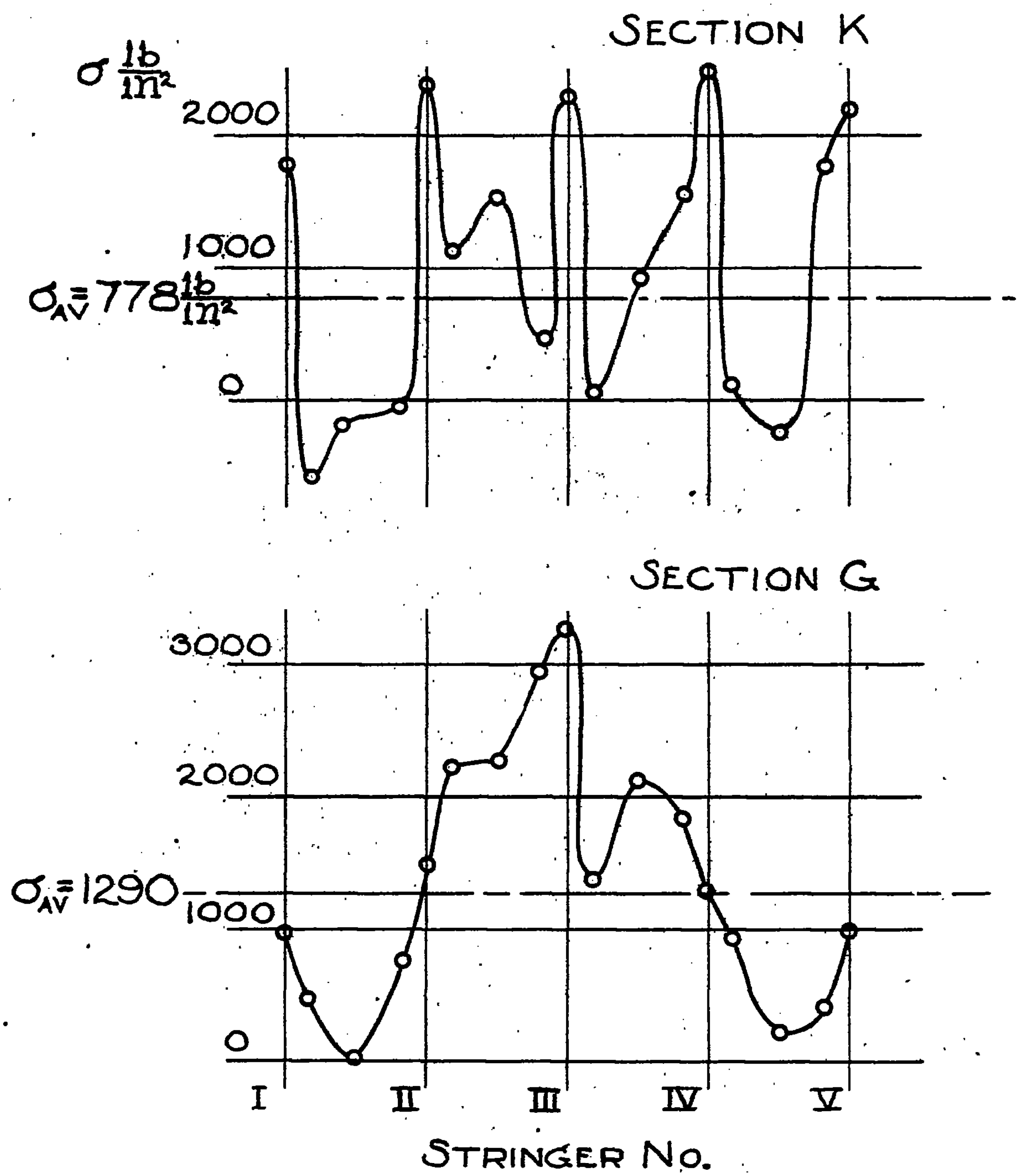


FIG. 11. DIRECT STRESS IN SHEET. 3000 LB LOAD CONDITION.

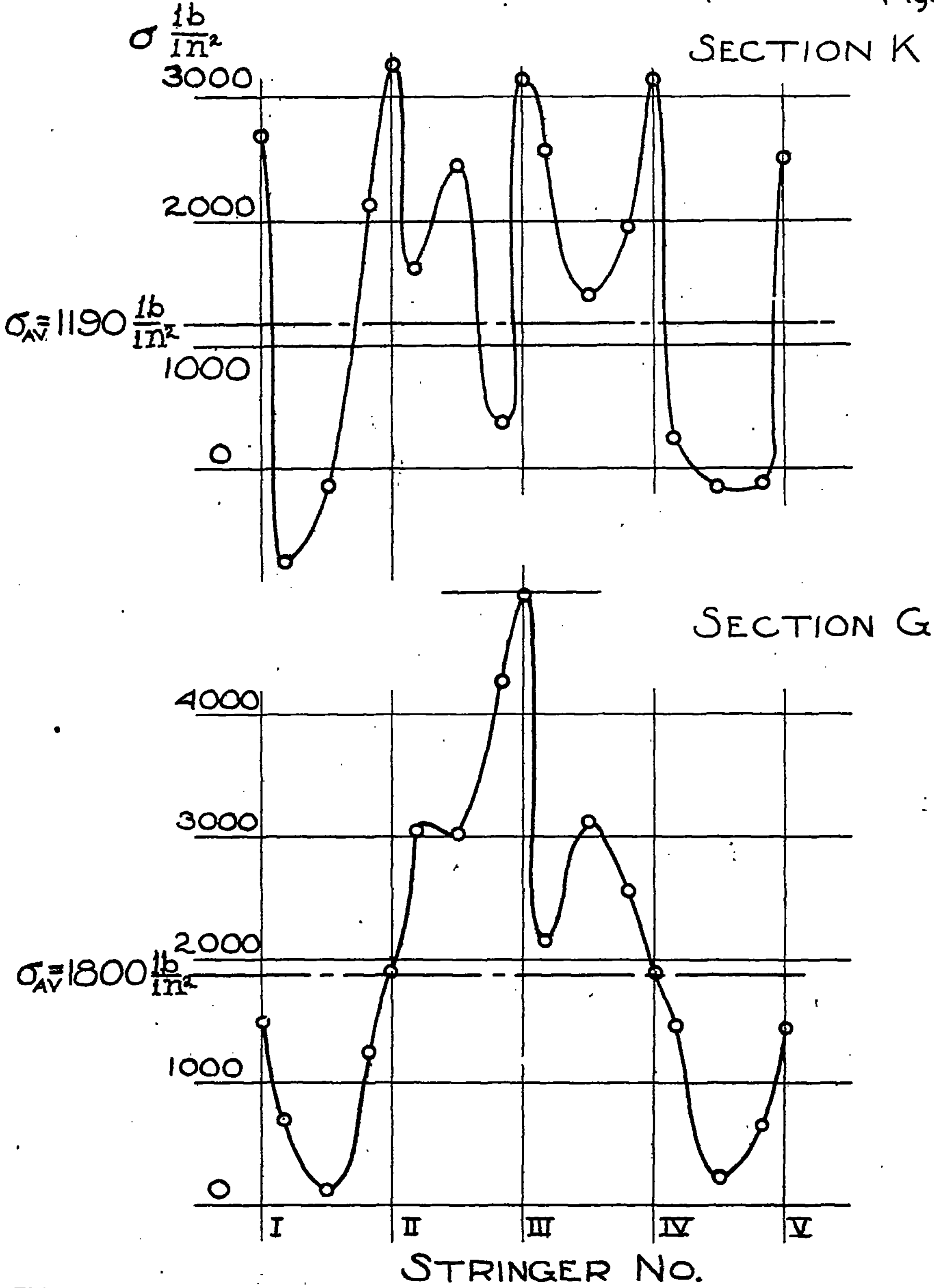


FIG. 12. DIRECT STRESS IN SHEET. 4500 LB LOAD CONDITION.

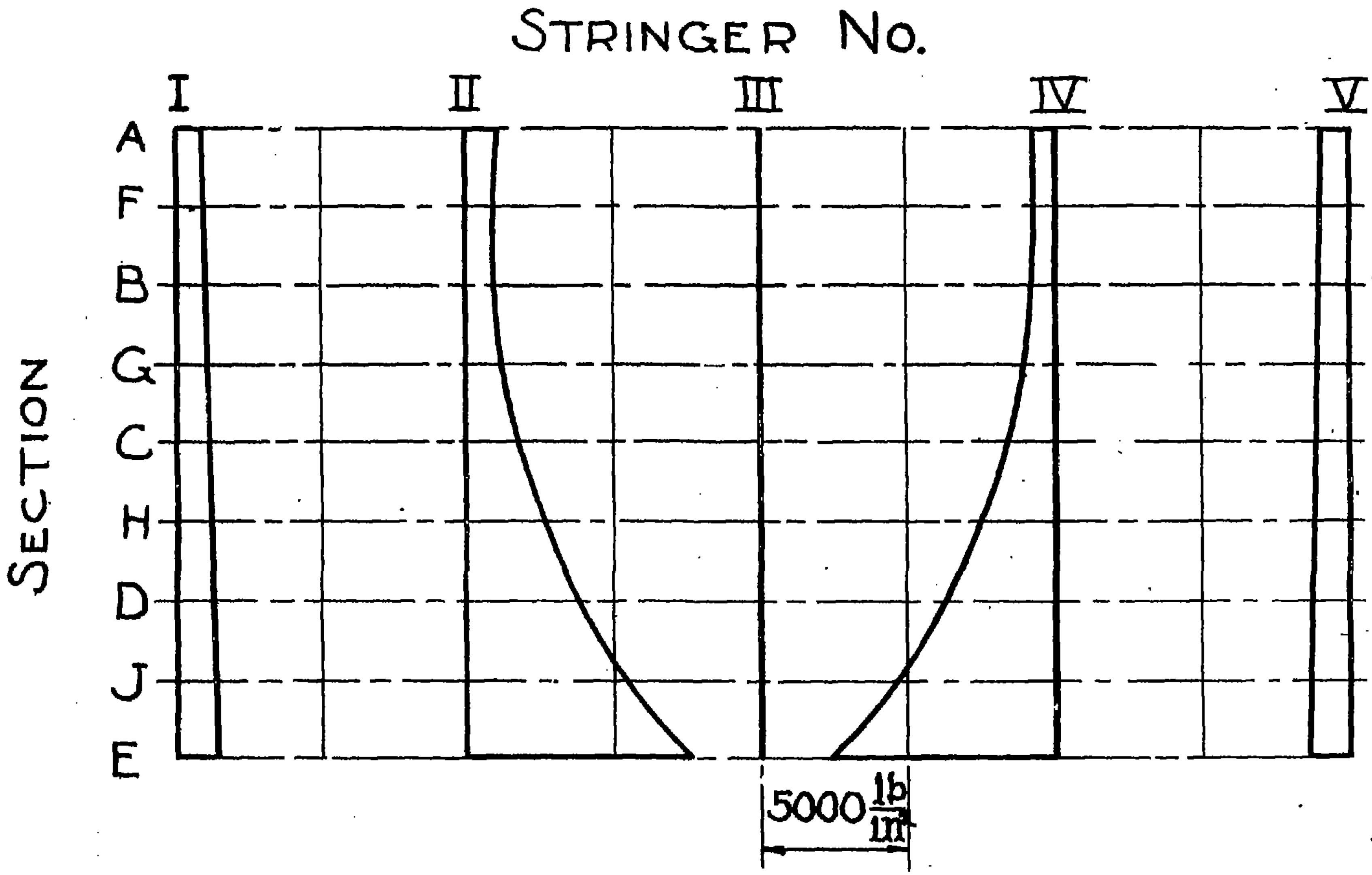


FIG. 13. SHEAR STRESS ALONG STRINGERS. 3000 LB LOAD CONDITION.

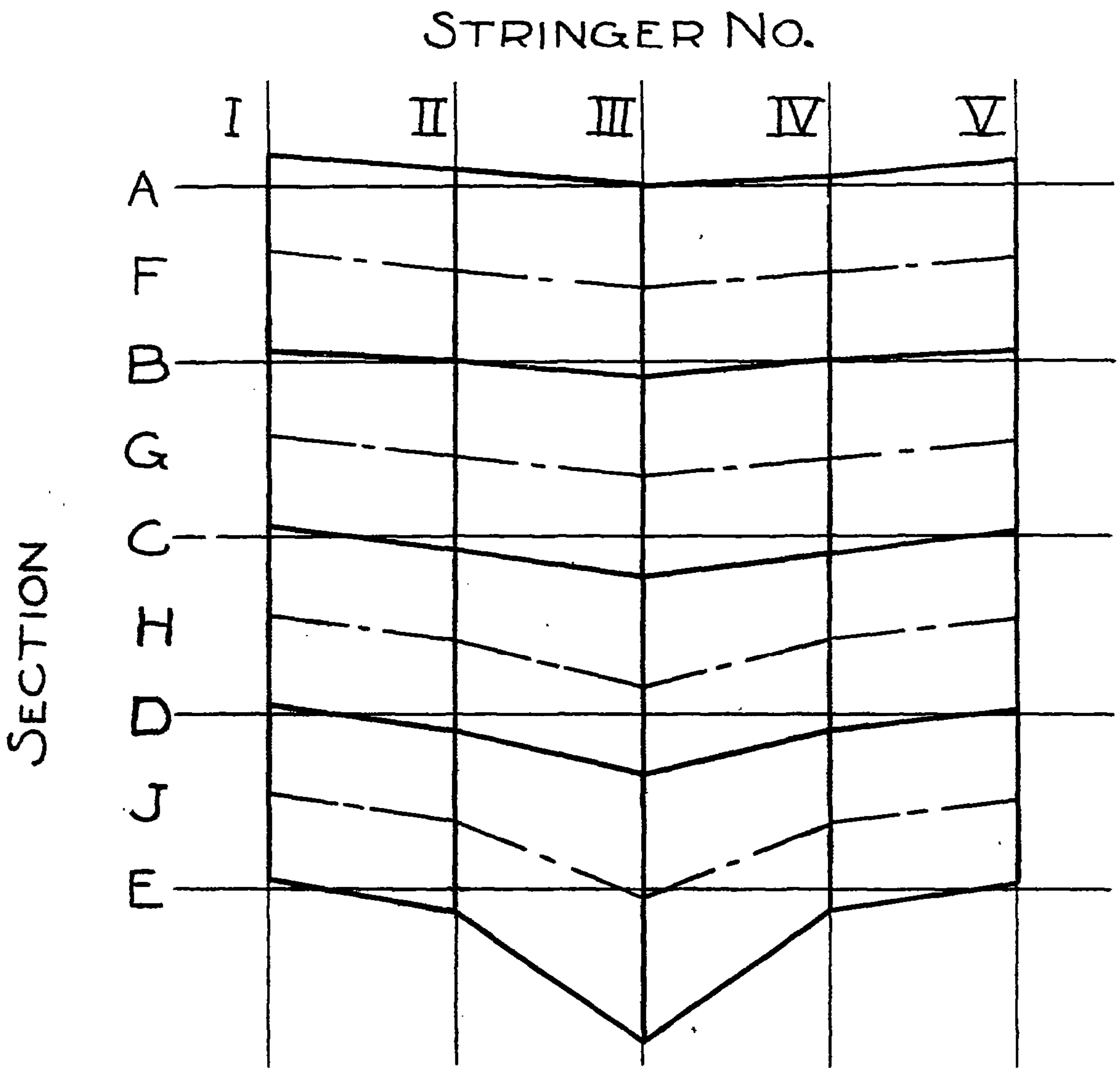


FIG. 14. DEFLECTED SHAPE OF MODEL.
3000 LB LOAD CONDITION.
SCALE: MODEL 1/6 DEFORMATION 200 TO 1.

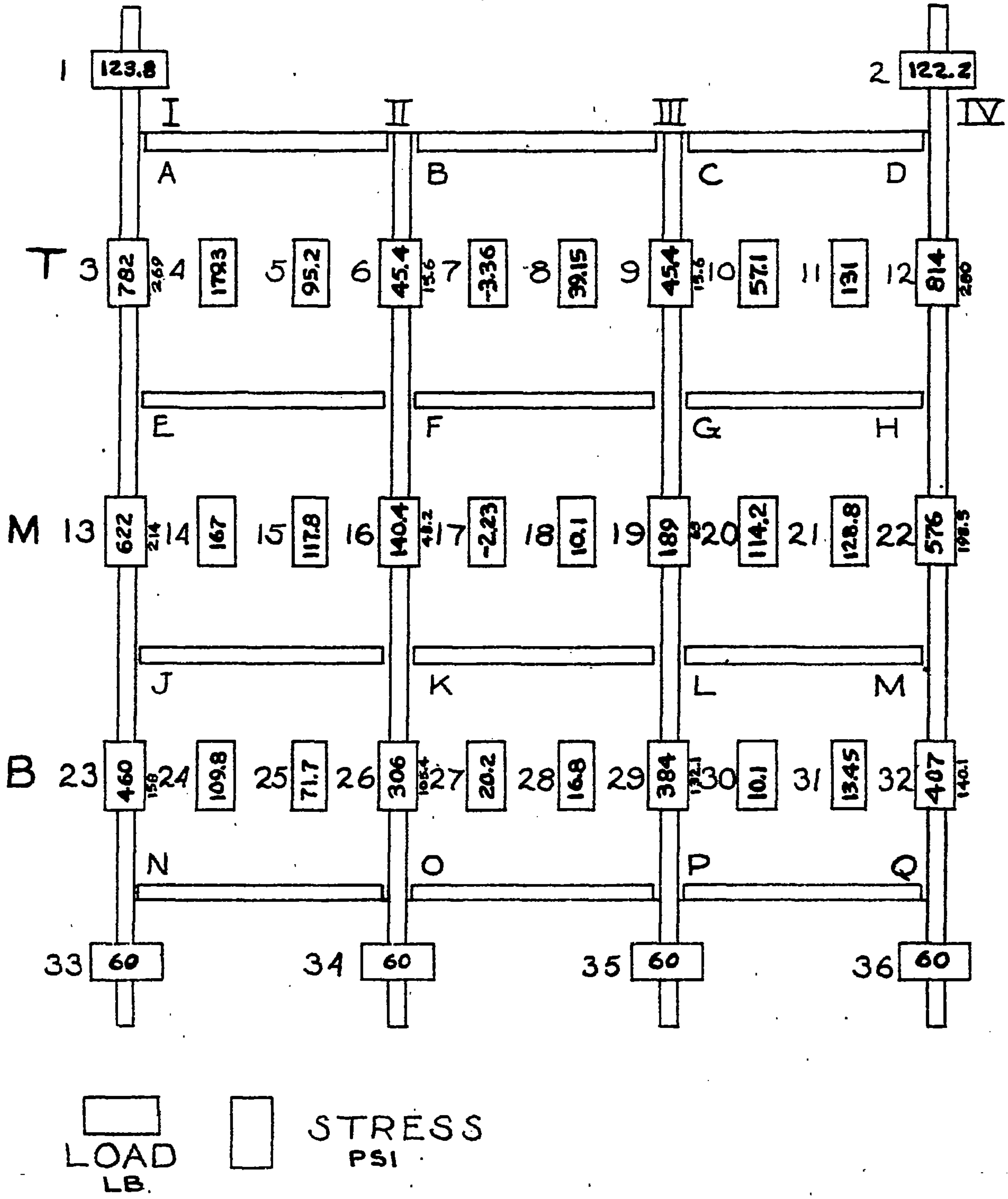


FIG. 15. FLAT MODEL WITH MEASURED LOADS AND STRESSES. 240 LB LOAD INCREMENT.

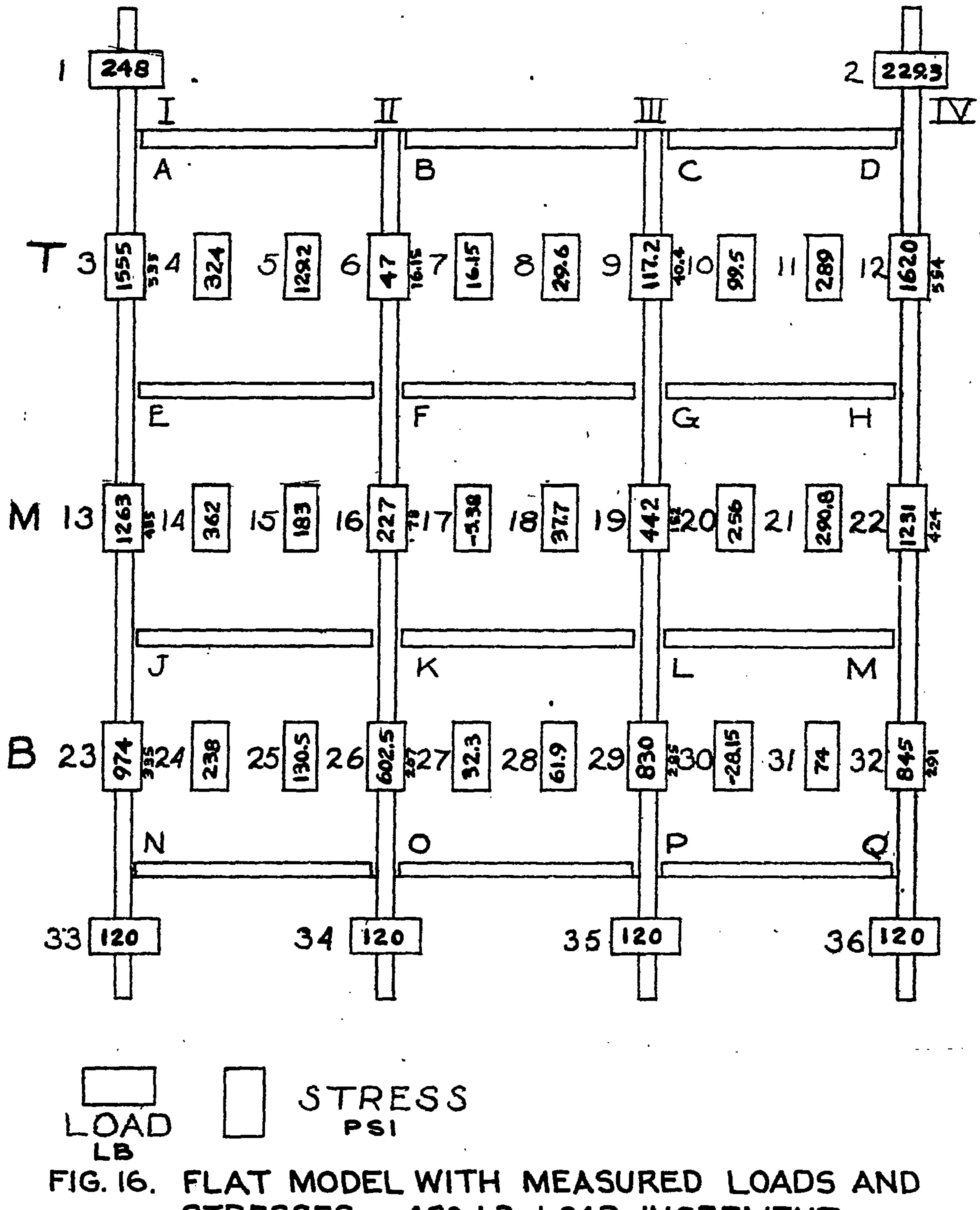


FIG. 16. FLAT MODEL WITH MEASURED LOADS AND STRESSES. 480 LB LOAD INCREMENT.

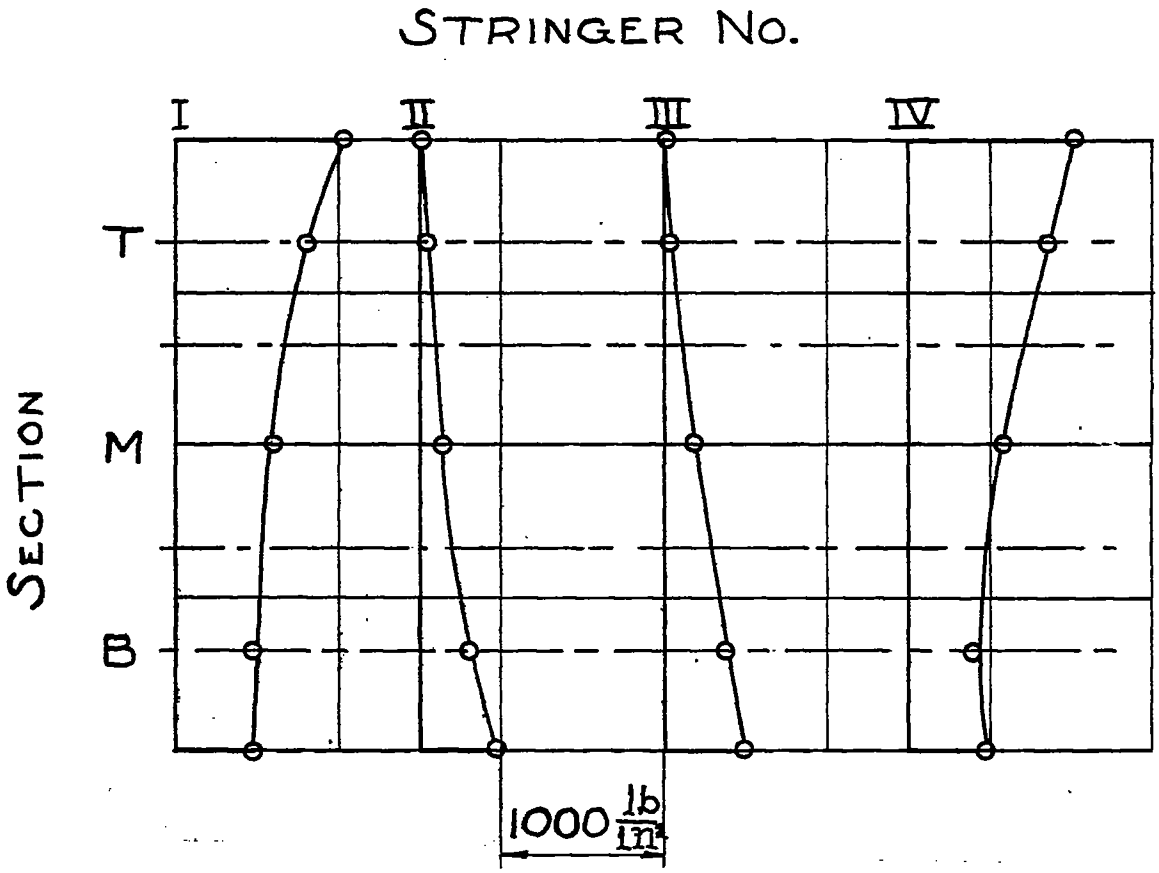


FIG. 17. DIRECT STRESS IN STRINGERS.
240 LB LOAD INCREMENT.

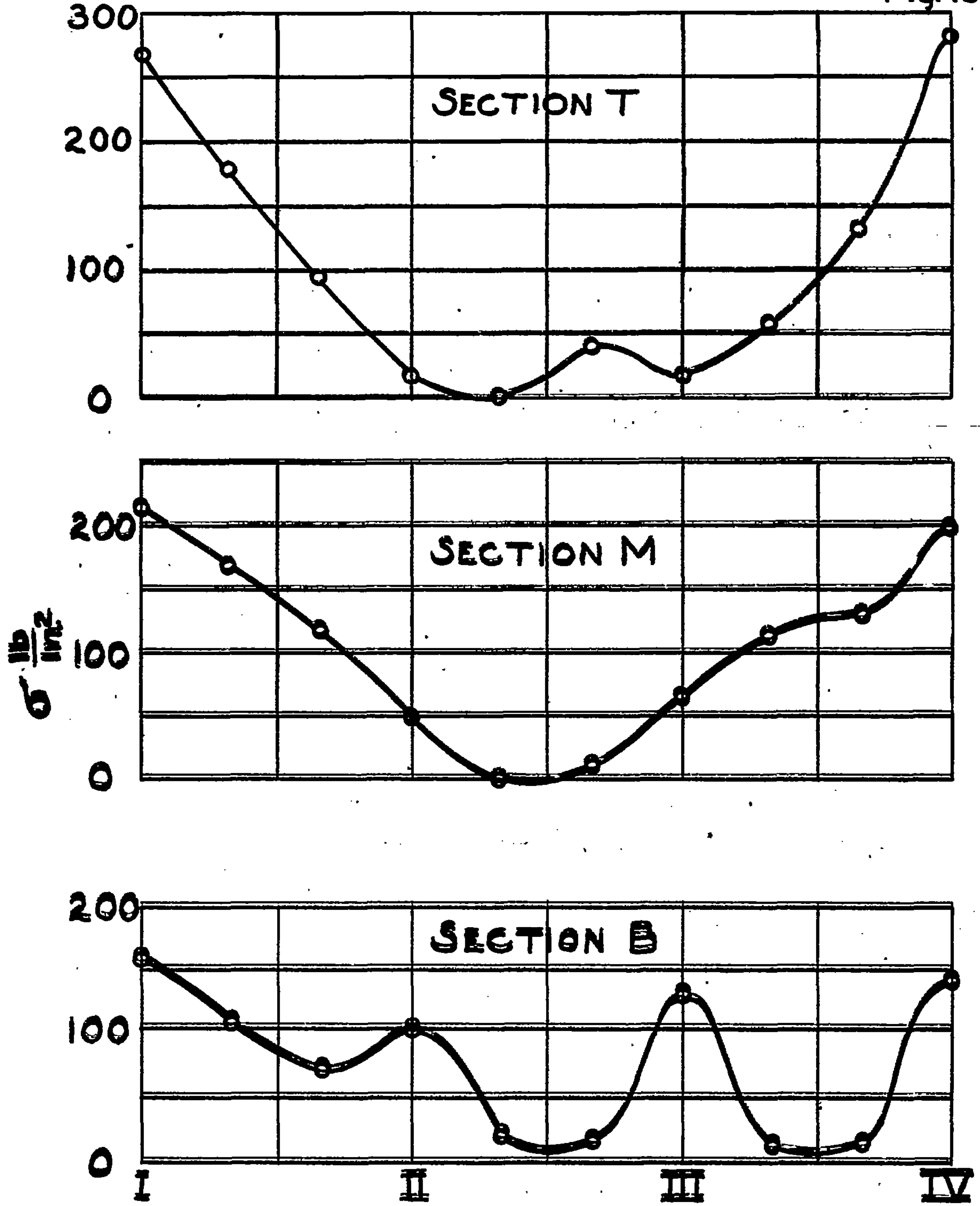
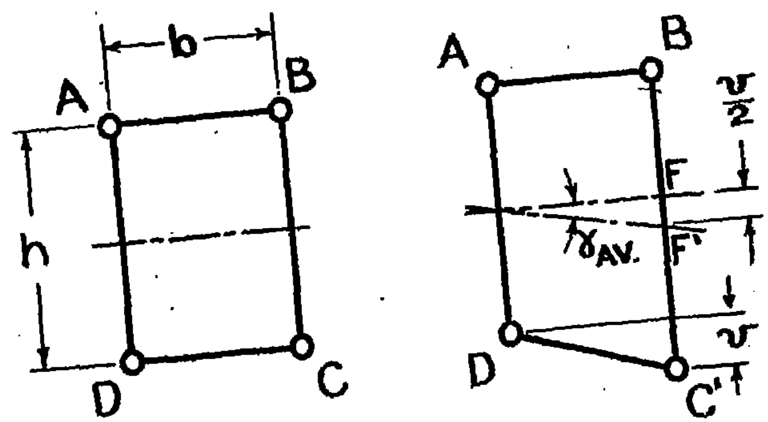


FIG. 18. DIRECT STRESS IN SHEET.
240 LB LOAD INCREMENT.



(a) (b)

FIG. 19. UNIT OF REINFORCED SHEET.

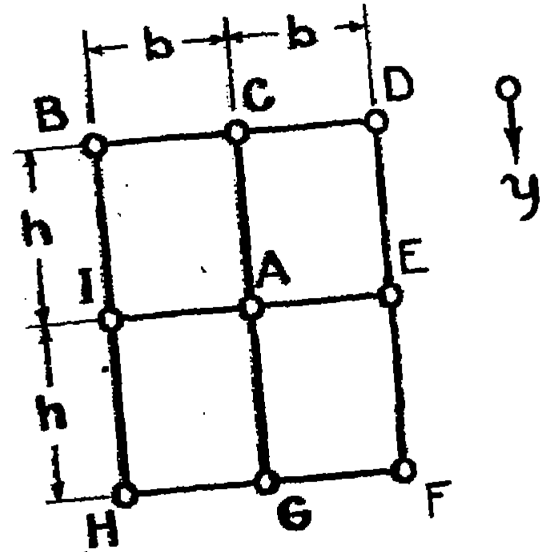


FIG. 20. FOUR PANEL SYSTEM.

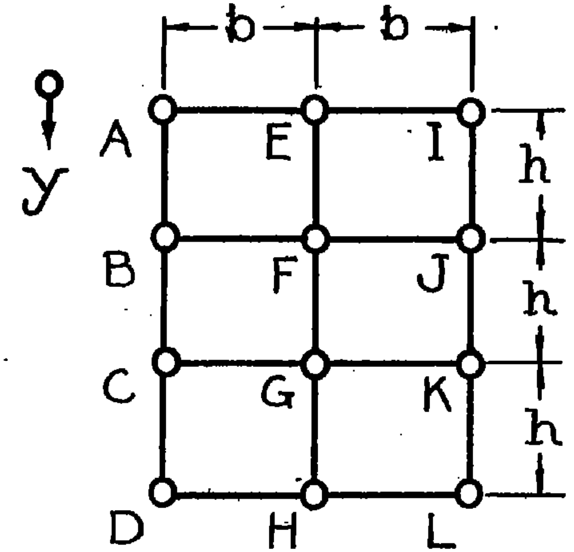


FIG. 21. SYSTEM FOR BLOCK DISPLACEMENT.

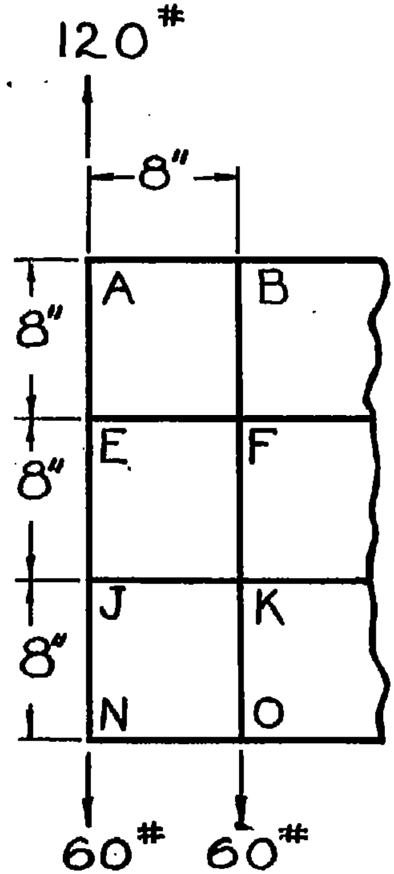


FIG. 22. SCHEMATIC DRAWING OF ONE-HALF THE FLAT MODEL.

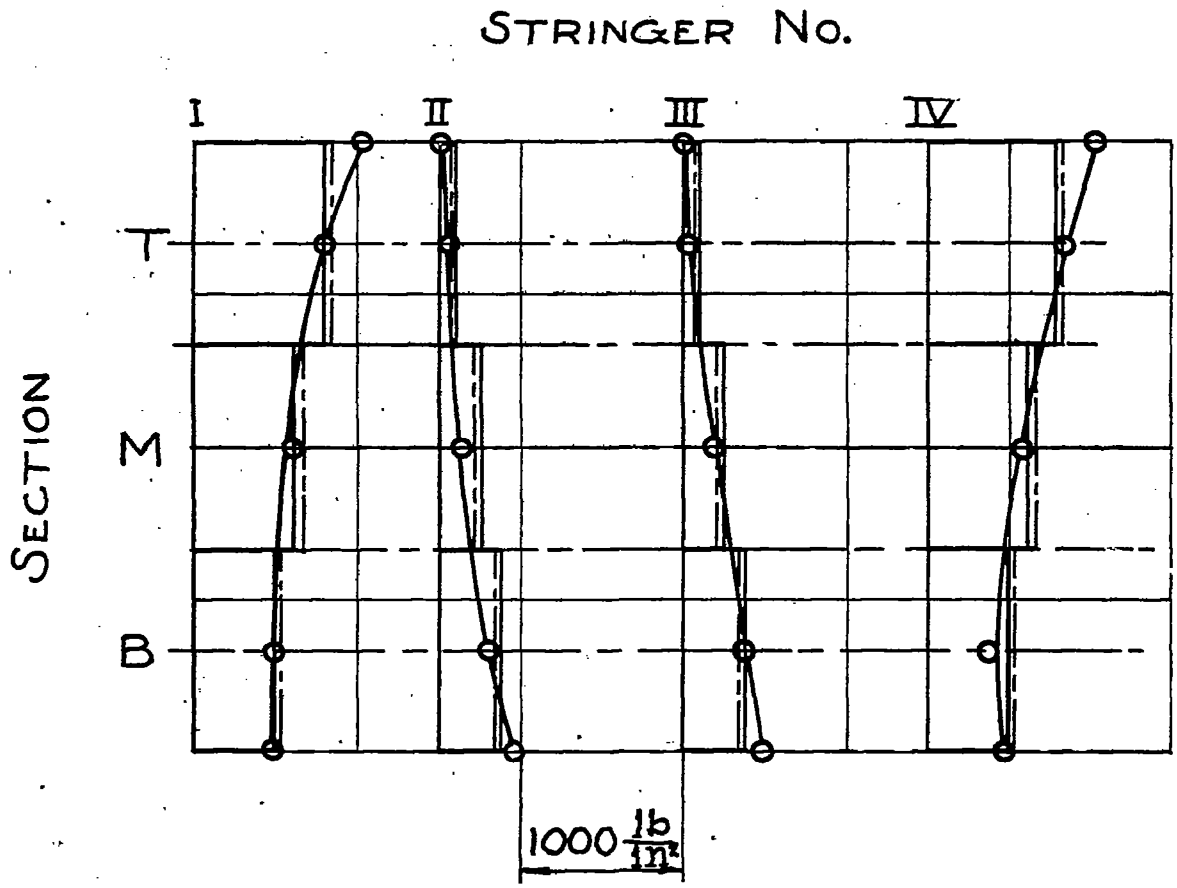


FIG. 23. CALCULATED AND MEASURED DIRECT STRESS IN STRINGERS IN FLAT MODEL.

o MEASURED VALUES.
——— CALCULATED, FULL SHEAR RIGIDITY.
—·— CALCULATED, ONE-QUARTER SHEAR RIGIDITY.

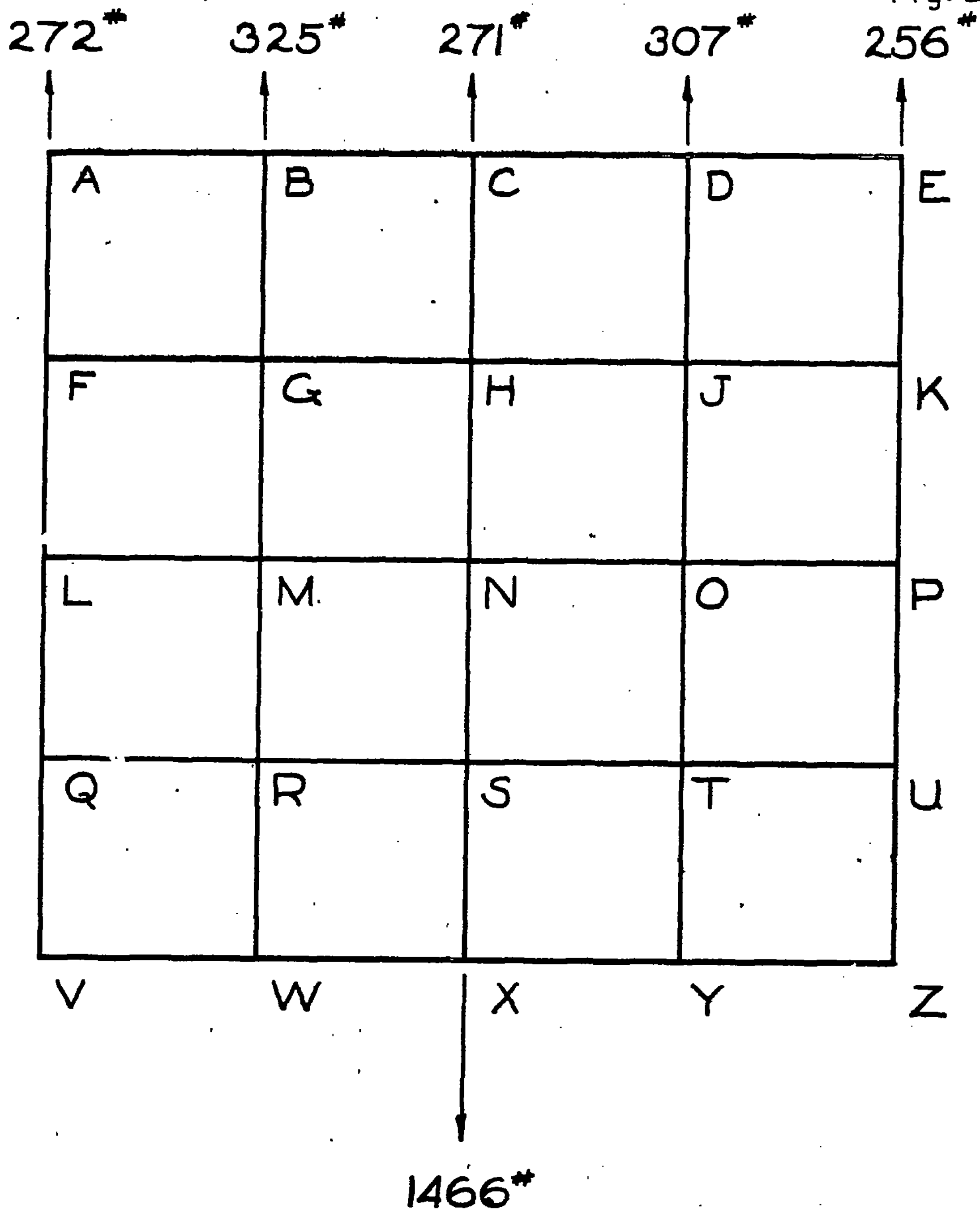


FIG. 24. SCHEMATIC DRAWING OF CURVED MODEL UNDER LOAD, RUN J.

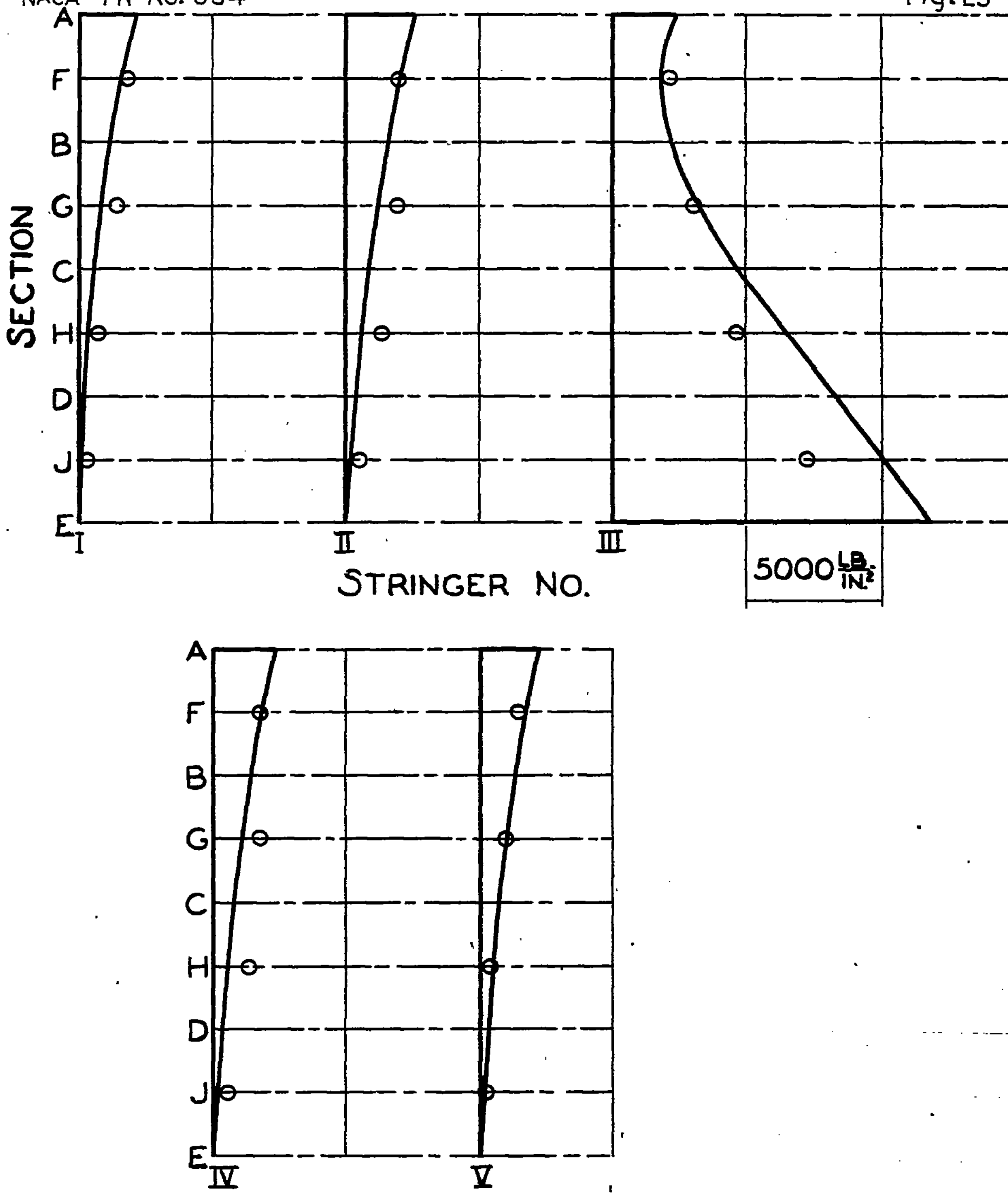


FIG. 25. CALCULATED AND MEASURED DIRECT STRESS IN STRINGERS FOR CURVED MODEL.
CURVES SHOW VALUES DERIVED FROM EXPERIMENT.
CIRCLES INDICATE CALCULATED VALUES.

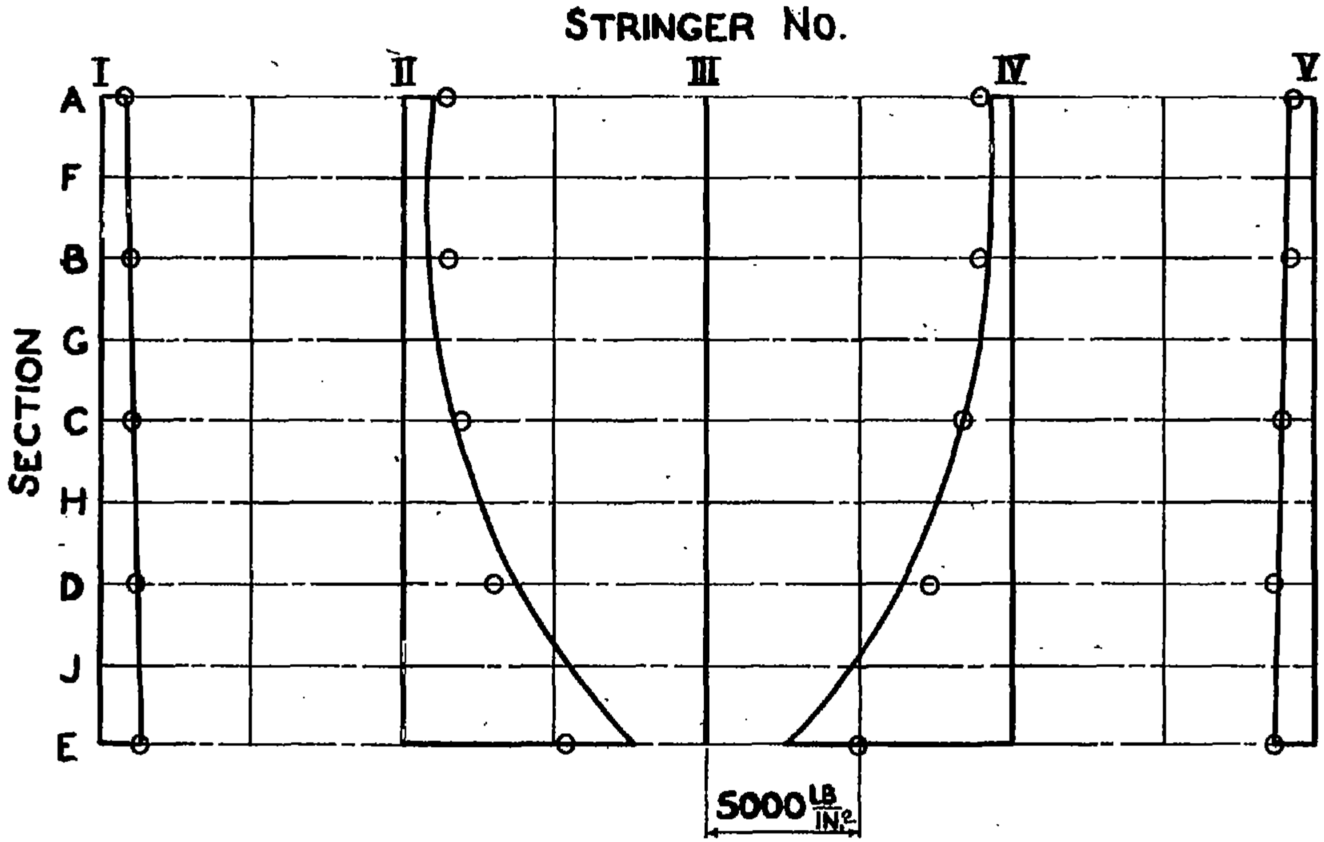


FIG. 26. COMPARISON OF SHEAR STRESSES FOR CURVED MODEL.
CURVES SHOW VALUES DERIVED FROM EXPERIMENT.
CIRCLES INDICATE CALCULATED VALUES.